Thales of Miletus Sources and Interpretations

Miletli Thales Kaynaklar ve Yorumlar

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Preface

Here are notes of what I have been able to find or figure out about Thales of Miletus. They may be useful for anybody interested in Thales. They are not an essay, though they may lead to one. I focus mainly on the ancient sources that we have, and on the *mathematics* of Thales.

I began this work in preparation to give one of several 20-minute talks at the Thales Meeting (Thales Buluşması) at the ruins of Miletus, now Milet, September 24, 2016. The talks were in Turkish; the audience were from the general population. I chose for my title "Thales as the originator of the concept of proof" (Kanıt kavramının öncüsü olarak Thales). An English draft is in an appendix.

The Thales Meeting was arranged by the Tourism Research Society (Turizm Araştırmaları Derneği, TURAD) and the office of the mayor of Didim. Part of Aydın province, the district of Didim encompasses the ancient cities of Priene and Miletus, along with the temple of Didyma. The temple was linked to Miletus, and Herodotus refers to it under the name of the family of priests, the Branchidae.

I first visited Priene, Didyma, and Miletus in 2008, when teaching at the Nesin Mathematics Village in Şirince, Selçuk, İzmir. The district of Selçuk contains also the ruins of Ephesus, home town of Heraclitus. In 2016, I drafted my Miletus talk in the Math Village. Since then, I have edited and added to these notes. Such work may continue, as my interest does.

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1. Introduction

The present work began on June 15, 2016, as a handwritten entry in a journal. I had just learned of the possibility of speaking about Thales of Miletus, *in* Miletus. Having looked up Thales in Heath's *History of Greek Mathematics* [42, 43] and Proclus's *Commentary on the First Book of Euclid's* Elements [66], and having remembered that Kant mentioned Thales in the Preface of the B Edition of the *Critique of Pure Reason* [50], I noted the attribution to Thales of the theorems that

1) a diameter bisects a circle, and
2) the base angles of an isosceles triangle are equal.

These theorems may seem obvious, from symmetry; but what if the symmetry is broken, by distorting the circle into an ellipse, or sliding to one side the apex of the triangle, as in Figure 1.1? Thales's theorems concern equality, which students today confuse with sameness. But Thomas Jefferson wrote that all of us were created equal; this does not mean we are all the same.

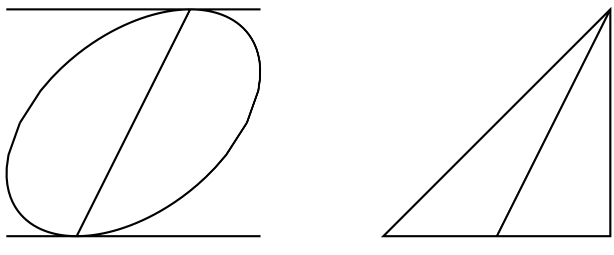


Figure 1.1.: Generalizations of Thales's theorems

Robert Recorde [68] invented the equals sign out of two *equal,* parallel (but distinct) straight lines. "If all men are created equal" I wrote,

> does this mean women too? And Africans? What about—animals? Citing Aristotle, Collingwood says Thales conceived of the *world* as an animal.

Equations are thought to scare people. Perhaps an equation like $A = \pi r^2$ *should* scare people: it is a modern summary of the most difficult theorem in Euclid.

Such were my early thoughts of what might be talked about on the theme of Thales. By August 15, my own talk had become definite. I would speak for 20 minutes in Miletus. I noted some additional possibilities. I had a memory from childhood of playing with a rubber band and wondering if, when unstretched, it always surrounded the same area, no matter what shape. I proved that it did not, since the shape surrounded by the rubber band could be collapsed to nothing. In my talk, I might display several shapes, all having the same perimeter, and ask whether they all had the same area. If the circle has the largest area, why is this? How can it be proved?

That the circle has maximal area among shapes with the same perimeter: I am as confident of this as of anything else in mathematics, and yet I do not know how to prove it. If I needed a proof, I would look up the calculus of variations; but then the question would remain: what is the use of a proof of a theorem that is obviously true?

To measure the area of a rectangle, you multiply the length by the width. If you need to measure the area of an arbitrary quadrilateral, you might think it reasonable to multiply the averages of the opposite sides. This is how the ancient Egyptians measured their fields, according to Fowler in *The*

Mathematics of Plato's Academy [34, §7.1(d), pp. 231–4; §8.1, pp. 279–81]; but the rule is not exact. Herodotus says the Greeks learned geometry from the Egyptians [45, ii.109]; and yet what we see in Book I of Euclid is how to measure a field with straight sides *exactly.*

Some of these thoughts made it into the talk that I actually gave in Miletus on September 24. The rest continued to be relevant, but there was only so much that could be said in 20 minutes. Meanwhile, I collected a good part of the notes below.

I do not recall how I decided to collect my notes into a single document; but using the electronic format seems the best way to edit, rewrite, and rearrange. The LaTeX program allows easy cross-referencing.

While the present document grew almost at random, I have tried to make it continuously readable. However, while it does have a beginning, it may not have a real middle or end. If there are ancient sources on Thales that have not been included in Chapter 2, I am not likely to be in a position to find them. I can go on looking up what modern philosophers and historians say about Thales. I *shall* go on thinking my own thoughts.

At this point, I do seem to have something new to suggest about Thales, and I said it in my talk. By saying all was water, Thales recognized a kind of unity in the world. Collingwood [17, pp. 29–30] and the Frankforts [35, p. 251] noted this (see pages 70 and 75 below). But I have not seen this unity connected to the unity of mathematics. A theorem is the recognition that a single principle can explain many observations. This recognition may well come before the possibility of *proving* the theorem. Merely asserting the theorem can be a great advance, comparable to the advance of explaining the world without appeal to gods. It looks as if Thales achieved *both* of these advances.

One can read the fragments of Heraclitus of Ephesus [44] as if they were a poem. They may be translated and presented *as* poetry [26]; they may be used in a new poem, as by T. S. Eliot in "Burnt Norton," the first of the *Four Quartets* [29, p. 117]. But the term "fragment" is misleading if it suggests (as it once did to me) random utterances. The fragments are quotations made by writers who knew Heraclitus's whole book. The serious scholar will read those writers, to get a sense of how they were reading Heraclitus.

We have no clear fragments of Thales himself, but we have ancient mentions of him. The mentions collected by Diels and Kranz [27] and by Kirk *& al.* [53] are often comparable in length to the fragments of Heraclitus. Again the serious scholar will read these mentions in context. I have provided some of this context in these notes, at least by quoting Aristotle at greater length. But still one should read more. One should study the other Milesians, Anaximander and Anaximenes. One should study the Egypt that Thales is said to have been visited.

There is no end to what one should study. This brings on a lament by Collingwood in *Speculum Mentis* [15, p. 231–2]. The chapters of this book are I Prologue, II Speculum Mentis, III Art, IV Religion, V Science, VI History, VII Philosophy, and VIII Speculum Speculi. In §5, "The Breakdown of History," in the History chapter, Collingwood observes that, since we cannot know everything, in history at least, it would seem that we cannot really know anything:

> History is the knowledge of the infinite world of facts. It is therefore itself an infinite world of thought: **history is essentially universal history, a whole in which the knowledge of every fact is included.**

> This whole, universal history, is never achieved. All history is fragmentary. The historian—he cannot help it—is a specialist, and no one takes all history for his province unless he is content to show everywhere an equal ignorance, and equal falsification of fact. But this is a fatal objection to the claims of historical thought as we have, without favour or exaggeration, stated them. History is the knowledge of an infinite whole whose parts, repeating the plan of the whole in their structure, are only known by reference to their context. But since this context is always incomplete, **we can never know a single part as it actually is.**
>
> . . . **If we are to escape this inference it can only be by** withdrawing everything we have said about the structure of historical fact and **substituting a new theory.**

The bold emphasis, as almost always throughout these notes, is mine. The theory that is supposed to solve the problem of history will be referred to by Collingwood as "historical atomism." I think he must be alluding to his contemporary Wittgenstein, whose *Tractatus Logico-Philosophicus* [79] had come out a few years earlier. In its own poetic or gnomic style, the *Tractatus* begins:

> 1 The world is everything that is the case.
>
> 1.1 The world is the totality of facts, not of things.
>
> . . .
>
> 1.2 The world divides into facts.
>
> 1.21 Any one can either be the case or not be the case, and everything else remain the same.
>
> 2 What is the case, the fact, is the existence of atomic facts.

This new theory does not work. It is only a declining to do history, since nothing studied by history happens in isolation. Collingwood continues:

> It is easy to see what this new theory will be. The individ-

> uality of historical facts, we must now say, is not systematic but atomic . . .
>
> . . . Now these atoms, as the word suggests, are nothing new in the history of thought. On the contrary, they represent a very old idea, and one from which the modern conception of critical history has but lately and with difficulty emerged. They are precisely the *instances* of an abstract law: that is to say, they belong, together with the allied notions of agglomeration or addition, infinite series, and externality, to the sphere of science. Historical atomism saves history by surrendering the whole thing and plunging back into the scientific consciousness. After recognizing this, we are the less surprised to see that its advocates are almost exclusively mathematicians. For it is simply a proposal to purge history of everything historical and reduce it to mathematics.

If Collingwood means to describe Wittgenstein and Russell as mathematicians, I don't know that the description fits. The mathematician as such is not *interested* in solving problems outside of mathematics.

Collingwood goes on to assert his own interest as an historian, on his pages 235–6:

> To translate our difficulty into empirical terms, we have already seen that periods of history thus individualized are necessarily beset by 'loose ends' and fallacies arising from ignorance or error of their context. Now there is and can be no limit to the extent to which a 'special history' may be falsified by these elements. The writer insists upon this difficulty not as a hostile and unsympathetic critic of historians, but as an historian himself, one who takes a special delight in historical research and enquiry; not only in the reading of history-books but in the attempt to solve problems which the writers of history-books do not attack. But

> as a specialist in one particular period he is acutely conscious that his ignorance of the antecedents of that period introduces a coefficient of error into his work of whose magnitude he can never be aware . . . Ancient history is easy not because its facts are certain but because we are at the mercy of Herodotus or some other writer, whose story we cannot check; contemporary history is unwritable because we know so much about it . . .

"Contemporary history is unwritable because we know so much about it." I first read this in a book whose first paragraph is a quotation of most of page 82 of *Speculum Mentis.* The book is *A Concise History of Modern Painting* [67], and here, in his second paragraph, Herbert Read quotes from page 236 as I am doing, before saying of himself,

> The present writer can claim to have given close and prolonged reflection to the facts that constitute the history of the modern movement in the arts of painting and sculpture, but he does not claim that he can observe any law in this history.

As I read Collingwood—and I agree—, it is *science* that is a search for laws; history is something else. What Collingwood concludes on his page 238 is that

> history is the crown and the *reductio ad absurdum* of all knowledge considered as knowledge of an objective reality independent of the knowing mind.

That is a grand claim, but one that I can agree with, while recognizing that it can be taken the wrong way. Thales had his thoughts, whether we know what they were or not. In this sense, those thoughts are, or were, an "objective reality." However, *as thoughts,* they can make no difference to us, unless we *do* manage to think them for ourselves; and doing this is entirely up to us. I think this is Collingwood's point. It is a point that he continued to sharpen over the course of his career, as in *The Principles of History* [22, p. 93] drafted fifteen years later:

> I do not doubt, again, that the purely physical effects produced in man's organism by its physical environment are accompanied by corresponding effects in his emotions and appetites; although this is a subject on which information is very difficult to procure, because what has been written about it has mostly been written by men who did not understand the difference between feelings and thoughts, or were doing their best, consciously or unconsciously, to obscure that difference. I will therefore take a hypothetical example. Suppose that there are two varieties of the human species in one of which sexual maturity, with all its emotional accompaniments, is reached earlier than in the other; and that this can be explained physiologically as an effect of climate and the like. Even so, this early sexual development is in itself no more a matter of historical interest than skin-pigmentation. It is not sexual appetite in itself, but man's thought about it, as expressed for example in his marriage-customs, that interests the historian.
>
> It is on these lines that we must criticize the all too familiar commonplaces about the 'influence' of natural environment on civilizations. It is not nature as such and in itself (where nature means the natural environment) that turns man's energies here in one direction, there in another: it is what man makes of nature by his enterprise, his dexterity, his ingenuity, or his lack of these things . . .

The distinction between thinking and feeling corresponds to the distinction between *The Principles of History* and Collingwood's earlier *Principles of Art* [16]. Collingwood was interested in both.

In *Speculum Mentis,* Collingwood's next section is called "The Transition from History to Philosophy." I pause to note an objection made by Leo Strauss [73, p. 563] to Collingwood's philosophy of history:

> He justly rejected Spengler's view that "there is no possible relation whatever between one culture and another." But he failed to consider the fact that there are cultures which have no actual relations with one another, and the implications of this fact: he dogmatically denied the possibility of "separate, discrete" cultures because it would destroy the dogmatically assumed "continuity of history" as universal history.

I shall only suggest that it is not very good criticism to complain that a writer "failed to consider" something. Perhaps he did consider it, but thought it not worth getting into. It might be something that the reader could work out for himself.

Strauss was reviewing *The Idea of History* [19], a posthumous publication that incorporated only parts of *The Principles of History.* One of those parts addresses a possible objection to the last quotation from that book:

> With the disappearance of historical naturalism, the conclusion is reached that the activity by which man builds himself his own constantly-changing historical world is a free activity . . .
>
> This does not mean that a man is free to do what he wants. All men, at some moments in their lives, are free to do what they want: to eat, being hungry, for example, or to sleep, being tired. But this has nothing to do with the freedom to which I have referred . . .
>
> Nor does it mean that a man is free to do what he chooses: that in the realm of history proper, as distinct from that of animal appetite, people are free to plan their own actions as they think fit and execute their plans, each doing what he set out to do and each assuming full responsibility for the

> consequences, captain of his soul and all that. Nothing could be more false . . .
>
> The rational activity which historians have to study is never free from compulsion: the compulsion to face the facts of its own situation . . . With regard to this situation he is not free at all . . .
>
> The freedom that there is in history consists in the fact that this compulsion is imposed upon the activity of human reason not by anything else, but by itself . . .

This is from pages 98–100 of *Principles,* and also pages 315–7 of *Idea,* though the beginning is different there. Collingwood will work more on the distinction between wanting and choosing in *New Leviathan* [21], particularly in chapters XI and XIII, "Desire" and "Choice," from which the following fragments are taken:

> **11. 1.** . . . In appetite or mere wanting a man does not know what he wants, or even that he wants anything; in desire or wishing, he not only knows that he wants something, but he knows what it is that he wants.
>
> . . .
>
> **13. 14.** Choice is not *preference,* though the two words are sometimes used as synonyms. *Preference is desire as involving alternatives.* A man who 'prefers' *a* to *b* does not choose at all; he suffers desire for *a* and aversion towards *b*, and goes where desire leads him.
>
> . . .
>
> **13. 2.** The problem of free will is not whether men are free (for every one is free who has reached the level of development that enables him to choose) but, how does a man become free? For he must be free before he can make a choice; consequently *no man can become free by choosing.*
>
> **13. 21.** The act of becoming free cannot be done to a man by anything other than himself. Let us call it, then, an

act of *self-liberation.* This act cannot be voluntary.

13. 22. ‘Liberation from what?’ From the dominance of desire. ‘Liberation to do what?’ To make decisions.

Starting in §2.4.2, page 32, I shall take up the theme of cultural continuity mentioned by Strauss. Meanwhile, I note the difficulty of knowing with any confidence what Thales thought. There are doubts that Collingwood’s thought was understood by his pupil, friend, and literary executor, Knox, the original editor of *The Idea of History,* who did not think most of *The Principles of History* worth publishing.

Why *try* to know somebody else’s thought? Thales is a legend, even today, particularly in the region of Miletus. This is why I found myself speaking about him. His legendary status may be used to provoke curiosity and thought.

Miletus and Ephesus were Ionian cities. They may still be a locus for thoughts worth thinking. Visiting the ruins of the Ionian cities on the west coast of Turkey in 1952, Freya Stark wrote [71, pp. 2–4]:

> Curiosity ought to increase as one gets older . . . Whatever it was, the Ionian curiosity gave a twist for ever to the rudder of time. It was the attribute of happiness and virtue. To look for the causes of it is a hopeless quest in Greece itself; the miracle appears there, perfect, finished and inexplicable. But in Asia Minor there may be a chance, where Thales of Miletus, ‘having learnt geometry from the Egyptians, was the first to inscribe a right-angled triangle in a circle, whereupon he sacrificed an ox.’
>
> . . . I am looking not for history but for happiness, a secret to be pursued with the accuracy of a different mood; and surely to be found; for—out of most hard and barbarous times, out of strangely modern vicissitudes, sacking of cities, emigration, slavery, exile—it still hangs unmistakable, elu-

> sive, like a sea-spray in the sun, over the coastline of Ionia.

Stark does go on to note the paradox of finding happiness in a place where the Persians might capture a city, kill most of the men, and enslave the women and children.

Collingwood finds happiness in medieval Europe, because of its institutions: the guild, the church, the monastic order, the feudal hierarchy. Writing in the Prologue of *Speculum Mentis* [15, p. 23], he does offer a disclaimer, as Stark does:

> we do not idealize medieval life or hold it free from defect. We do not forget either the corruptions to which these institutions too often succumbed, their tendency to level downwards, or the hideous fate of those adventurous souls who found their limits too narrow. But the very tendency to level downwards, the very narrowness of medieval institutionalism, secured one great benefit, namely the happiness of those humble ordinary men and women who ask not for adventure or excitement, but for a place in the world where they shall feel themselves usefully and congenially employed.

Ionia *is* a place for adventure. In writing of Clazomenae, Stark notes how Anaxagoras left it and settled in Athens, possibly even having travelled in the train of Xerxes. However, in the end,

> Athens, different and old-fashioned, was outraged by Anaxagoras who announced the sun to be a red-hot stone and the moon made of earth with hills and valleys, and who spoke in a friendly way, perhaps, of the easy-going Persian rule, under which his youth had grown; so that he was brought up for trial, and rescued by Pericles, who seems to have smuggled him out of prison and away. He came back to the liberal atmosphere of Ionia, not to Clazomenae but to Lampsacus, a colony from Miletus, where he taught and died, and asked that school-children be given an annual holiday to remember

> him by when he was dead. This was still done many years later, and the citizens also put up in their market-place, in his memory, an altar to Mind and Truth.

In the present work I seek happiness *through* history. Freya Stark quotes Diogenes's fanciful story about how Thales performed a sacrifice after proving a theorem. I consider the theorem itself in §2.8, page 54. In the Nesin Mathematics Village near Ephesus in the summer of 2016, I collaborated successfully to prove a couple of theorems, and I studied Thales. It might be pleasant to establish a ritual upon proving a theorem; but there is little reason to think that Thales actually sacrificed an ox on such an occasion. I want to know Thales, as far as possible, as he is, and not just as legend makes him to be.

2. Sources

2.1. A legend from Diogenes Laertius

In the *Lives of Eminent Philosophers,* Diogenes Laertius (3rd century C.E.) devotes to Thales the second chapter of Book I, the chapter comprising ¶¶22–44 of the book. In ¶24, Diogenes mentions Pamphila as attributing one theorem to Thales. I shall make use of Diogenes chiefly for this attribution (see §2.8, page 54) and for the story [28, I.27–9] to be quoted presently. I quote the story because it mentions Didyma along with Miletus, and because it is an example of how Diogenes is not a critical historian. In preparing the present notes though, I may be like Diogenes in passing along uncritically what I have found; my first concern is just to note what is out there about Thales.

The Wise Men mentioned at the beginning of Diogenes's story are listed in ¶13 as (1) Thales, (2) Solon, (3) Periander, (4) Cleobulus, (5) Chilon, (6) Bias, and (7) Pittacus; but variations are given at ¶¶40–2.

> The well-known story of the tripod found by the fishermen and sent by the people of Miletus to all the Wise Men in succession runs as follows. Certain Ionian youths having purchased of the Milesian fishermen their catch of fish, a dispute arose over the tripod which had formed part of the catch. Finally the Milesians referred the question to Delphi, and the god gave an oracle in this form:

> Who shall possess the tripod? Thus replies
> Apollo: "Whosoever is most wise."
>
> Accordingly they give it to Thales, and he to another, and so on till it comes to Solon, who, with the remark that the god was the most wise, sent it off to Delphi. Callimachus in his *Iambics* has a different version of the story, which he took from Maeandrius of Miletus. It is that Bathycles, an Arcadian, left at his death a bowl with the solemn injunction that it "should be given to him who had done most good by his wisdom." So it was given to Thales, went the round of all the sages, and came back to Thales again. And he sent it to Apollo at Didyma, with this dedication, according to Callimachus:
>
> Lord of the folk of Neleus' line,
> Thales, of Greeks adjudged most wise,
> Brings to thy Didymaean shrine
> His offering, a twice-won prize.
>
> But the prose inscription is:
>
> > Thales the Milesian, son of Examyas [dedicates this] to Delphinian Apollo after twice winning the prize from all the Greeks.
>
> The bowl was carried from place to place by the son of Bathycles, whose name was Thyrion, so it is stated by Eleusis in his work *On Achilles,* and Alexo the Myndian in the ninth book of his *Legends.*

Diogenes writes in ¶40, after describing the death of Thales,

> To him belongs the proverb "Know thyself" (*Γνῶθι σαυτόν*) which Antisthenes in his *Successions of Philosophers* attributes to Phemonoë, though admitting that it was appropriated by Chilon.

There is little reason to lend credence to any of this, except perhaps insofar as it reflects what people really did believe about Thales. We may however be interested to note also (from ¶25) that Thales is supposed to have saved the Milesians from an alliance with the Lydians, who were soon to be conquered by the Persians:

> Thales is also credited with having given excellent advice on political matters. For instance, when Croesus sent to Miletus offering terms of alliance, he frustrated the plan; and this proved the salvation of the city when Cyrus obtained the victory. Heraclides makes Thales himself say [in a dialogue] that he had always lived in solitude as a private individual and kept aloof from State affairs.

Finally, at the end of the chapter (¶44), Diogenes quote a letter in which Thales is supposed to have invited Solon to live in Miletus or Priene:

> If you leave Athens, it seems to me that you could most conveniently set up your abode at Miletus, which is an Athenian colony; for there you incur no risk. If you are vexed at the thought that we are governed by a tyrant, hating as you do all absolute rulers, you would at least enjoy the society of your friends. Bias wrote inviting you to Priene; and if you prefer the town of Priene for a residence, I myself will come and live with you.

2.2. Kirk, Raven, and Schofield

Thales wrote no book, except possibly one called *Nautical Star-guide,* and even the authorship of this was disputed in ancient times. My source here is Chapter II, "Thales of Miletus," of Kirk, Raven, and Schofield, *The Presocratic Philosophers* [53, pp. 76–99]. The whole book is based on 616 quotations of ancient authors. Given in the original language (Greek or Latin) and in English translation, the quotations are numbered in boldface, serially throughout (which is why I can say how many there are). The quotations are of three orders, not indicated by the numbering: (1) the main quotations, taking the full width of the text body (only their translations are indented); (2) quotations given in the commentary on the main quotations; and (3) quotations given in footnotes (they are numbered like footnotes, but printed at the ends of sections rather than pages).

Chapter I of Kirk *& al.* is "The Forerunners of Philosophical Cosmogony." The next five chapters concern "The Ionian Thinkers," namely (1) Thales, (2) Anaximander, and (3) Anaximenes, all of Miletus; (4) Xenophanes of Colophon; and (5) Heraclitus of Ephesus. Colophon and Ephesus are now in İzmir province.

The chief sources on Thales are Herodotus, Aristotle, and Proclus. The last lived in the fifth century, a century after the founding of Constantinople, and thus, by some reckonings, in the Middle Ages (the year dividing McEvedy's atlases of ancient and medieval history [57, 56] is 362). Other sources are Diogenes Laertius and a few others, notably Plato.

A summary of the Thales chapter of Kirk *& al.* may be in order: I give it here by section (with abbreviated title) and number of quotation. Express quotations are from Kirk *& al.* themselves or their translations, unless I give another reference. The parenthetical citations are of quotations that are not the main ones in the sense above. Citations **62–68** are **63–69** in the first edition of the book, by Kirk and Raven only [52]. Missing from the second edition is **70** in the first:

> Plutarch *de Is. et Osir.* 34, 364D. "They think that Homer also, like Thales, made water principle and birth of all things through learning from the Egyptians."

Then citations **69–93** are **71–95** in the first edition. If Kirk *& al.* give an explicit reference to one of the quotations of Diels and Kranz (DK) listed in the next section, I give the reference too. Kirk *&* *al.* have additional references to Diels and Kranz in their commentary.

Nationality. Thales is said to be Phoenician, but was probably "as Greek as most Milesians."

62 Diogenes I, 22 (DK 11A1 *init.*).
63 Herodotus I, 170 (from **65**).
64 (Herodotus I, 146.)

Activities. See §2.6, page 39.

65 Herodotus I, 170. The bouleuterion at Teos.
66 Herodotus I, 75. The crossing of the Halys.

Egypt. He is said to have visited, and this is probably true.

67 Aetius I, 3, 1. "Thales . . . having practiced philosophy in Egypt came to Miletus when he was older."
68 Proclus *in Euclidem* p. 65 Friedl. (from Eudemus) (DK 11A11).
69 (Herodotus II, 109, on the Egyptian origin of geometry *quâ* surveying.)
70 Herodotus II, 20, on the Nile flooding; see §2.6.3, page 41
71 Aetius IV, 1, 1. "Thales thinks that the Etesian winds . . . ," as in **70**.

Typical philosopher. "Neither of these stories is likely to be strictly historical."

72 Plato, *Theaetetus* 174A. He was mocked by a Thracian servant girl for falling into a well while looking at the stars.

73 Aristotle, *Politics* A11, 1259a9. Mocked for being poor, he studied the heavens, predicted a bumper crop of olives, and rented all the olive presses, thus making a killing at harvest time. (Diogenes tells the story briefly at I.26, attributing it to Hieronymos of Rhodes.)

Astronomy. See §2.6, page 39.

74 Herodotus I, 74. Thales predicted the solar eclipse of 585.

75 Diogenes I, 23. "He seems by some accounts to have been the first to study astronomy, the first to predict eclipses of the sun and to fix the solstices; so Eudemus in his *History of Astronomy*" [28].

76 Dercyllides *ap.* Theon Smyrn. p. 198, 14 Hiller. "Eudemus relates in the *Astronomy* that Thales ⟨first discovered⟩ the eclipse of the sun and the variable period of its solstices."

77 (Herodotus I, 29. Greek sages, including Solon, visited Sardis under Croesus.)

78 Callimachus *Iambus* I, 52, fr. 191 Pfeiffer (DK 11A3a). Thales "measured out the little stars of the Wain, by which the Phoenicians sail."

Mathematics.

79 Diogenes I, 27. "Hieronymus says that he [Thales] actually measured the pyramids by their shadow, having observed the time when our own shadow is equal to our height."

80 Proclus *in Euclidem* p. 352 Friedl. (DK 11A20). See §2.7, page 42.

Writings. See the head of the section.

81 Simplicius *Phys.* p. 23, 29 Diels.
82 Diogenes I, 23.
83 Suda s.v. (from Hesychius) (DK 11A2).

Cosmology. See below (mainly §2.9, page 58).

(i) The earth floats on water, the source of all things.

84 Aristotle, *De Caelo* B13, 294a28. The earth rests on water.

85 Aristotle, *Met.* A3, 983b6. The first principle is water.

86 (Aristotle, *De Anima* A2, 405b1. Hippo said the soul was water.)

87 (Heraclitus Homericus *Quaest. Hom.* 22. " . . . Thales declared that water, of the four elements, was the most active, as it were, as a cause.")

88 Seneca *Qu. Nat.* III, 14 (presumably from Theophrastus, through a Posidonian source). "For he [Thales] said that the world is held up by water and rides like a ship, and when it is said to 'quake' it is actually rocking because of the water's movement."

(ii) The inanimate can be alive; the world is full of gods.

89 Aristotle, *de an.* A2, 405a19. The magnet has a soul.

90 Diogenes I, 24 (similarly; the quotation is in §2.5, page 38, because Collingwood uses it).

91 Aristotle, *de an.* A5, 411a7: all things are full of gods.

92 (Plato, *Laws* 10, 899A. "Is there anyone who will accept this and maintain that all things are *not* full of gods?")

93 (Aetius I, 7, 11. "Thales said that the mind of the world is god, and that the sum of things is besouled, and full of daimons; right through the elemental moisture there penetrates a divine power that moves it.")

2.3. Diels and Kranz

Diels's *Fragmente der Vorsokratiker,* revised by Kranz [27], is apparently a comprehensive collection of all writings, remaining from ancient times, that are by or about somebody involved with Greek philosophy before Socrates. However, I have found ancient sources on Thales not given by Diels and Kranz: see §2.4.

The "Presocratic Philosophy" article in the *Stanford Encyclopedia of Philosophy* [25] provides a useful guide for the uninitiated (such as myself):

> The standard collection of texts for the Presocratics is that by H. Diels revised by W. Kranz (abbreviated as DK). In it, each thinker is assigned an identifying chapter number (e.g., Heraclitus is 22, Anaxagoras 59); then the reports from ancient authors about that thinker's life and thought are collected in a section of "testimonies" (A) and numbered in order, while the passages the editors take to be direct quotations are collected and numbered in a section of "fragments" (B). Alleged imitations in later authors are sometimes added in a section labeled C. Thus, each piece of text can be uniquely identified: DK 59B12.3 identifies line 3 of Anaxagoras fragment 12; DK 22A1 identifies testimonium 1

on Heraclitus.

In the old days it might have been assumed that interested scholars could figure these things out for themselves, if only because they would know enough German to read the prefatory material.

Here I list the citations given by Diels and Kranz concerning Thales. I transcribe them by cutting and pasting (and correcting and marking up, as needed) from the `djvu` file that I have. As far as I can tell, all main quotations of Kirk *& al.* appear here, except **67** and **71,** which are of Aetius. Trying to find the Kirk *& al.* quotations here caused me to realize, for example, that Diels and Kranz's 22 really had *two* quotations from *De Anima.* There could be such oversights still undetected; I cannot always understand Diels and Kranz's notation.

A. LEBEN UND LEHRE

1 Diogenes Laertius I 22–44. The whole chapter on Thales. KRS **62, 75, 79, 82, 90**

2 Suidas [Z. 25–30 aus Hesychios Onomatologos . . .] KRS **83**

3 Schol. Platonis in remp. 600A [aus Hesych]

3a Callimach. Iamb. [fr. 94 . . .]. KRS **78**

4 Herodot. I 170. KRS **63, 65**

5 — I 74. KRS **74**

6 Herod. I 75. KRS **66**

7 Euseb. Chron.

8 Ἐκλογὴ Ἱστοριῶν Parisina

9 Plato Theaet. 174 A. KRS **72**

10 Aristot. Pol. A. 11 1259a 6 KRS **73**

11 Procl. in Eucl. 65, 3 Friedl. KRS **68**

11a Himer. 30 Cod. Neap. [Schenkl *Herm.* 46, 1911, 420]

12 ARISTOT. Metaphys. A 3. 983b 6. KRS **85**
13 SIMPL. Phys. 23, 21
13a AËT. I 17, 1 (D. 315)
13b — II 1, 2 (D. 327)
13c — II 12, 1 (D. 340)
14 ARIST. de caelo B 13. 294a 28. KRS **84**
15 SENECA Nat. Quaest. III 14. KRS **88**
16 HEROD. II 20. KRS **70**
17 DERCYLLIDES ap. Theon. astr. 198, 14 H. KRS **76**
17a AËT. II 13, 1 (D. 341)
17b — II 27, 5 (D. 358)
18 PLIN. N. H. XVIII 213
19 APULEIUS Flor. 18 p. 37, 10 Helm
20 PROCL. in Eucl.

- 157, 10 Friedl. (aus Eudem)
- 250, 20
- 299, 1
- 352, 14. KRS **80**

21
- PLIN. N. H. XXXVI 82.
- PLUT. Conv. VII sap. 2 p. 147 A.

22 ARIST. de anima A 5. 411a 7. A 2. 405a 19. KRS **91, 89**
22a AËT. IV 2, 1 (Dox. 386a, 10)
23 AËT. I 7, 11 (D. 301). KRS **93**
APOPHTHEGMATIK Vgl. Diog. § 35ff. I 71, 10 und c. 10, 2. 3δ I 64, 1.

B. ANGEBLICHE FRAGMENTE

ΘΑΛΟΥ ΝΑΥΤΙΚΗ ΑΣΤΡΟΛΟΓΙΑ

1 DIOG. I 23. SUID. SIMPL. Phys. 23, 29. KRS **81–3**
2 SCHOL. ARAT. 172 p. 369, 24 (Hyaden)

ΠΕΡΙ ΑΡΧΩΝ ΑΒ

3 GALEN. in Hipp. de hum. I 1 [XVI 37 K.]

ΠΕΡΙ ΤΡΟΠΗΣ. ΠΕΡΙ ΙΣΗ ΜΕΡΙ ΑΣ

4 Diog. i 23

2.4. Additional sources

There are ancient sources not mentioned, or at least not made official quotations by Kirk *& al.* or Diels and Kranz. At least I thought this was true of the Plutarch; but then it turned out to be the second part of DK 11A21.

2.4.1. Plutarch and "Thales's Theorem"

According to the English *Wikipedia,* Thales's Theorem is that the angle inscribed in a semicircle is right. In the Turkish Vikipedi, that theorem is given in the article Thales teoremi (çember), while the Thales teoremi is basically Euclid's Proposition VI.2, that a straight line cutting two sides of a triangle cuts them proportionally if and only if it is parallel to the base. This naming is confirmed in a Turkish test preparation book [51, p. 45]:

> **Thales Teoremi:** Paralel doğruların kendilerini kesen doğrular üzerinde ayırdıkları parçalar karşılıklı olarak orantılıdır.

This is basically the theorem of similar triangles discussed in §2.7.5. The English *Wikipedia* describes this theorem under the title the Intercept Theorem, while acknowledging the term Thales's Theorem, which is traced to a passage from Plutarch's *Dinner of the Seven Wise Men* [64, §2, pp. 351–3]:

> Thales began to laugh, and said, "If it is anything bad, go to Priene again! For Bias will have a solution for this, just as he had his own solution of the first problem."
>
> "What," said I, "was the first problem?"

"The king," said he, "sent to Bias an animal for sacrifice, with instructions to take out and send back to him the worst and best portion of the meat. And our friend's neat and clever solution was, to take out the tongue and send it to him, with the result that he is now manifestly in high repute and esteem."

"Not for this alone," said Neiloxenus, "but he does not try to avoid, as the rest of you do, being a friend of kings and being called such. In your case, for instance, the king finds much to admire in you, and in particular he was immensely pleased with your method of measuring the pyramid, because, without making any ado or asking for any instrument, you simply set your walking-stick upright at the edge of the shadow which the pyramid cast, and, two triangles being formed by the intercepting of the sun's rays, **you demonstrated that the height of the pyramid bore the same relation to the length of the stick as the one shadow to the other.** But, as I said, you have been unjustly accused of having an animosity against kings, and certain offensive pronouncements of yours regarding despots have been reported to him. For example, he was told that, when you were asked by Molpagoras the Ionian what was the most paradoxical thing you had ever seen, you replied, 'A despot that lived to be old.' And again he was told that on a certain convivial occasion there was a discussion about animals, and you maintained that of the wild animals the worst was the despot, and of the tame the flatterer.

Diogenes [28, 1.36] too mentions Thales's aged despot, without naming a source.

2.4.2. Iamblichus

Van der Waerden [78, p. 88] mentions skeptically the account of Iamblichus in *The Life of Pythagoras* [48, pp. 8–9], according to which Thales advised Pythagoras to go study in Egypt:

> When about eighteen years of age, fearing that his studies might be interfered with under the tyranny of Policrates [*sic*], he [Pythagoras] departed privately by night with Hermodamas, surnamed Creophilus; then he went to Pherecydes and to Anaxamander [*sic*], the natural philosopher, and also he visited Thales at Miletus. All of these teachers admired his natural endowments and imparted to him their doctrines. **Thales, after teaching him such disciplines as he possessed, exhorted his pupil to sail to Egypt** and associate with the Memphian and Diospolitan priests of Jupiter by whom he himself had been instructed, giving the assurance that he would thus become the wisest and most divine of men. Thales also taught him to be sparing of his time; hence he entirely abstained from wine and animal food, confining himself to such nourishment as was slender and easy of digestion; his sleep was short, his soul vigilant and pure, his body in a state of perfect and invariable health.

While saying that Iamblichus's chronology is off, van der Waerden suggest that there is a kernel of truth:

> it seems quite possible that Pythagoras learnt geometry from Egyptian priests and arithmetic from the Babylonian magi. In Egypt, he may have had contact with the "rope-stretchers", who were experts in geometrical constructions and proofs, and in Babylon he may have acquainted himself with Babylonian arithmetic (including the calculation of Pythagorean triples) and algebra. This would explain the many points of contact between Babylonian algebra and

Greek geometric algebra.

In reviewing van der Waerden's book, Fletcher [33] describes its thesis as that a lot of mathematics has a common origin in central Europe:

> The ancient civilisations of the title include Egyptian, Babylonian, Greek, Chinese and Indian. There is no surprise so far. But what of the civilisations in England (and Scotland) around about 2500 BC? And even more to the point, what was happening mathematically in central Europe, perhaps 500 years before that? The author examines the mathematical legacies from these seemingly disparate civilisations, discovers many areas of similarity, and puts forward the hypothesis of a common origin in central Europe from which the knowledge spread out to the UK, to Egypt, Babylonia, India, China and Greece. In fact, he goes further than this by adopting a "hypothesis of dependence" whenever great discoveries are found to be widely known. His first example is the theorem of Pythagoras and the ensuing Pythagorean triples, and this is a very powerful example indeed.

This sense of the book is confirmed in van der Waerden's own Introduction. I shall only suggest that, if humanity should contact intelligences from another planet, their mathematics will be the easiest to understand of all their works of thought; and this will not lend any credence to theories of "Chariots of the Gods." Mathematics does not spread like a virus, or grow within oneself like a cancer; it is a free creation of the intelligent will.

On the other hand, I may recreate within my own mind the theorem taught me by another. This is a theme of Collingwood, taken up in the next section.

2.5. Collingwood

I am going to list here the passages about Thales that Collingwood cites in *The Idea of Nature.* First though, continuing a thought provoked by van der Waerden, I observe that, if we know anything from Thales today, we know it because Thales was a teacher, as were the people who passed along his knowledge. Writing in the *New Leviathan* [21, ch. XXXVI] in response to the Nazis, Collingwood finds the essence of civilization in our civility to one another, a civility that involves sharing what we know about how to make use of the natural world:

> **36. 33.** If a community has attained any degree, high or low, of civlization relatively to the natural world, it is by acquiring and conserving an incredible amount of natural science. Partly, no doubt, by improving on it; but in this kind of science **improving on what is handed down to us is far less important than conserving it;** a fact which it is well to remember.
>
> **36. 34.** The proportion between the two things has been much misunderstood in the last century or two when for accidental and temporary reasons Europeans have attached too much importance to invention and too little to conservation.

In seeking the essence of civilization, Collingwood has alluded to Thales:

> **36. 19.** According to the traditional logic a definition must state the essence of the term to be defined; and the essence of a term (in this case the term civilization) can only involve one differentia.
>
> **36. 20.** Where there are two differentiae (e.g. 'an isosceles triangle has two sides equal *and two angles equal*') the definition is faulty; one of the two ought to be shown to follow as a necessary consequence from the other.

See §2.7.3 (page 46) and §3.1 (page 67).

Collingwood was a sailor, from his childhood on Coniston Water in the Lake District [49] to a cruise of the Mediterranean with Oxford undergraduates in 1939 at the age of 50 [23]. In the *New Leviathan,* he goes on to discuss knots—which by the way are a subject of mathematical theory, but this is not Collingwood's concern. Though each knot was invented by single person, thousands of years ago, our ability to benefit from those knots depends on the willingness of countless generations to share the skill of tying them.

> **36. 46.** The mainspring of the whole process is the spirit of agreement. So vast a body of knowledge (I call it knowledge, but it is not the kind of thing logicians call knowledge; it is all practical knowledge, knowing how to tie a bowline, knowing how to swim, knowing how to help a lambing ewe, how to tickle a trout, where to pitch a tent, when to plough and when to sow and when to harvest your crop) can only be brought together in a community (for it is too vast for the mind of one man) whose custom it is that everybody who has anything to teach to anyone else who wants to know it shall teach it; and that everybody who does not know a thing that may be useful for the betterment of living shall go frankly to one who knows it, confident by custom of a civil answer to a civil question.

In his section on Thales and Pythagoras, van der Waerden quotes from near the beginning of Aristotle's *Metaphysics* [8, I.i.16, $981^{\mathrm{b}}24$]:

> Thus the mathematical sciences originated in the neighbourhood of Egypt, because there the priestly class was allowed leisure.

Collingwood quotes the very first sentence of the *Metaphysics* ($980^{\mathrm{a}}22$) and enlarges on it:

36. 62. . . . The passion for learning and the passion for teaching have not disappeared from humanity. They still live.

36. 63. It is as true as when Aristotle wrote it that all men have a natural desire for knowledge.

36. 64. It is true, too, that **all men have a natural desire to impart knowledge.**

36. 65. That there is also a desire, at war with this, to gain power over men by monopolizing knowledge I do not deny.

36. 66. But **although there is certainly an eristic of knowledge,** a tendency to make it a matter of contention and competition and monopoly, **there is also a dialectic of knowledge,** a tendency to make it a matter of agreement and co-operation and sharing.

Apparently Collingwood himself was witness to the rise of an eristic of knowledge through the "professionalization of philosophy." This is a sectional head in the chapter on F. H. Bradley in Stephen Trombley, *Fifty Thinkers Who Shaped the Modern World* [77, p. 115]:

> . . . In the period between 1850 and 1903 there wasn't a school of British *idealism,* there was simply *British* philosophy, the general tendency of which was idealist. 'British idealism' is better regarded as a pejorative term created by early analytic philosophers to identify the status quo they wished to supplant with their own brand of thinking. The strange death of idealism in British philosophy goes hand in hand with philosophy's transformation from a gentleman's pastime into a profession . . . [T. H.] Green's career is a milestone in the history of philosophy because, according to the utilitarian Henry Sidgwick (1838–1900), he was the first *professional* philosopher in the English-speaking world.

The early analytic philosophers' war on British idealism

> can be seen to involve much more than the desire to supplant neo-Hegelian idealism and metaphysics in its entirety with logicism: they also wanted the idealists' jobs. The analytic side won both battles. The professionalization of philosophy in Britain and the United States resulted in the death of idealism and the erection of analytic philosophy as the official way of thinking; in this way a generation of teachers led by Russell, Moore and Wittgenstein spawned a new generation of followers, who in turn kept the analytic torch burning brightly in the English-speaking world throughout the twentieth century as their students and their students' students took up university teaching jobs. (There are notable exceptions to analytic dominance . . .)

Collingwood's admiration for the "school of Green," and his disdain for its destroyers, is seen in *An Autobiography* [18]; his distaste for labels like "idealist" is seen in the "Correspondence between R. G. Collingwood and Gilbert Ryle" appended to the new edition of *An Essay on Philosophical Method* [24]. A dig at positivism if not analytic philosophy is found in the long quotation of Frankfort and Frankfort from *Before Philosophy* in §3.3 (page 74).

In §3.2 (page 69) I consider what Collingwood has to say about Thales in *The Idea of Nature* [17]. Not all of his sources are in Kirk *& al.*, though they may then be in Diels and Kranz [27]. Here I list all ancient sources cited by Collingwood, according to the page number of the footnote in which they are cited.

Page 30

- Diogenes Laertius. KRS **82.**
- Theophrastus. Attribution to Thales of "a work on astronomy for sailors."
- Galen. DK B3. "The treatise 'on Beginnings' which

Galen quotes was certainly a forgery."

- "By Aristotle's time it was a matter of conjecture what his cosmological doctrines were" (Collingwood's remark).

Page 31

- Aristotle. KRS 85, DK A12.

Page 32

- Diogenes I, 24 (KRS **90**, [28]):

  > *Ἀριστοτέλης δὲ καὶ Ἱππίας φασὶν αὐτὸν καὶ τοῖς ψύχοις μεταδιδόναι ψυχῆς, τεκμαιρόνενον ἐκ τῆς λίθου τῆς μαγνήτιδος καὶ τοῦ ἠλέκτρου.* // Aristotle and Hippias affirm that, arguing from the magnet and from amber, he attributed a soul or life even to inanimate objects.

- Diogenes I, 27 [28]:

  > *Ἀρχὴν δὲ τῶν πάντων ὕδωρ ὑπεστήσατο, καὶ τὸν κόσμον ἔμψυχον καὶ δαιμόνων πλήρη.* // His doctrine was that water is the universal primary substance, and that the world is animate and full of divinities.

- Aristotle. KRS **89**
- Aristotle. KRS **84**
- Diogenes [28, ¶35, pp. 36–7]. (See also §4.6, page 91 below.)

  > Here too are certain current apophthegms (*ἀποφθέγματα*) assigned to him:
  >
  > > Of all things that are, the most ancient is God, for he is uncreated.
  > > *πρεσβύτατον τῶν ὄντων θεός· ἀγένητον γάρ.*
  > > The most beautiful is the universe, for it is God's workmanship.

κάλλιστον κόσμος· ποίημα γὰρ θεοῦ.

- An extrapolation:

  > That the earth 'grazes' on water is not a doctrine anywhere expressed in the fragments of Thales or ascribed to him by any ancient writer, but I am not alone in thinking it implied in the recorded fragments and their context. 'Le monde des choses est donc au milieu de l'eau et *s'en nourrit'* (A. Rey, *La jeunesse de la Science grecque,* Paris, 1933, p. 40: my italics).

2.6. Herodotus

The quotations by Kirk *& al.* of Herodotus that actually mention Thales are from Book I, chapters 74, 75, and 170. These are the only references to Thales in the index of Strassler's edition of Herodotus [72], and they tell us the following.

2.6.1. Solar eclipse (74)

Thales predicted the year of a solar eclipse, which occurred in the sixth year of war between the Lydians and the Medes; the two parties subsequently made peace. Strassler gives the date of eclipse as May 28, 585. This is apparently the Julian date; Guthrie in *A History of Greek Philosophy* [37, p. 46] notes that the Gregorian date is May 22. One of Guthrie's references happens to be in my possession: Heath, *Aristarchus* [41], where the Julian date is given in note 3, page 15. As an example of historical detective work, I quote from this note. Heath mentions here references to the eclipse in Cicero and Pliny and also in

> Eusebius, *Chron. (Hieron.),* under the year of Abraham 1433, 'An eclipse of the sun, the occurrence of which Thales had predicted: a battle between Alyattes and Astyages'. The eclipse so foretold is now most generally taken to be that which took place on the (Julian) 28th May, 585. A difficulty formerly felt in regard to this date seems now to have been removed. Herodotus (followed by Clement) says that the eclipse took place during a battle between Alyattes and Cyaxares. Now, on the usual assumption, based on Herodotus's chronological data, that Cyaxares reigned from about 635 to 595, the eclipse of 585 B.C. must have taken place during the reign of his son; and perhaps it was the knowledge of this fact which made Eusebius say that the battle was between Alyattes and Astyages. But it appears that Herodotus's reckoning was affected by an error on his part in taking the fall of the Median kingdom to be coincident with Cyrus's accession to the throne of Persia, and that Cyaxares really reigned from 624 to 584, and Astyages from 584 to 550 B.C. . . . ; hence the eclipse of 585 B.C. would after all come in Cyaxares's reign. Of two more solar eclipses which took place in the reign of Cyaxares one is ruled out, that of 597 B.C., because it took place at sunrise, which would not agree with Herodotus's story. The other was on 30th September, 610, and, as regards this, Bailly and Oltmanns showed that it was not total on the presumed field of battle (in Cappadocia) . . .

Kirk *& al.* give only the year 585, and only at the beginning of their chapter, not where they quote Herodotus; but there they surmise that Thales used the records of the Babylonians, kept since 721 B.C.E. Citing **77**, as above, Kirk *& al.* seem to suggest that the Babylonian records were available at Sardis. This does not make much sense, since the eclipse would have happened before the fall of Croesus, even before the rise, while

Sardis was still Lydian.

2.6.2. The crossing of the Halys (75)

Thales helped the army of Croesus cross the Halys River by diverting it around them, according to the Greeks; Herodotus thinks the army used the existing bridges.

2.6.3. The bouleuterion at Teos (170)

Thales of Miletus was Phoenician by descent, and he recommended that the Ionians have a single deliberative chamber [53] or council house [72], that is, a bouleuterion (*βουλευτήριον* [45]), centrally located in Teos. In *Aegean Turkey,* Bean's account of the ruins of Teos [13, pp. 106–15] mentions no bouleuterion, although his index (under Council House) lists one for each of Heracleia, Miletus, Notium, and Priene. Nonetheless, what Bean calls the odeon of Teos is labelled as a bouleuterion at the site itself (according to my blog article [62] recording a visit in May of 2015, and confirmed by another visit in September, 2016).

Kirk *& al.* also quote Aetius (**71**) as attributing to Thales the theory that the flooding of the Nile is caused by the Etesian winds ("The regular N.W. winds which blow in summer from the Mediterranean" [45, p. 299, n. 1]); Herodotus states the theory (without naming Thales) at II.20 (quotation **70**).

Herodotus concludes his *Histories* with the failed invasion of Greece by Xerxes begun in 480, more than a century after the 585 eclipse whose prediction Herodotus attributes to Thales. Presumably these two men were not alive at the same time. Strassler gives the years of Croesus's reign at Sardis as 560–547/6 and notes that Herodotus lived to see the Peloponnesian

War, begun in 431 [72, p. x]. Pericles's funeral oration from the first year of the war is quoted in §4.4, page 87.

2.7. Proclus

Kirk *& al.* are not elaborate in their use of Proclus. According to the index of Morrow's edition [66], there are five mentions of Thales in Proclus's *Commentary on the First Book of Euclid's* Elements. The lines of the Friedlein edition [65] are 65.7, 157.11, 250.20, 299.4, and 352.15. Morrow gives the last four in a footnote at the first place; he there also suggests as references (1) Heath, *A History of Greek Mathematics* [42, pp. 130–7]; (2) Gow, *History of Greek Mathematics* [36, pp. 138–45]; and (3) Van der Waerden, *Science Awakening* (New York, 1961) 85–90. I have and shall be using the first two. As for van der Waerden, I have got hold of his later *Geometry and algebra in ancient civilizations,* which devotes two pages [78, pp. 87–8] to Thales and Pythagoras. In §2.4.2 I discussed these pages, because they concern a source (Iamblichus) not found in Kirk *& al.* or Diels and Kranz. Van der Waerden tempers his earlier skepticism that the Egyptians could have taught the Greeks geometry.

Proclus's source on Thales seems to be the now-lost history of mathematics by Eudemus of Rhodes, pupil of Aristotle. Morrow refers to Heath, who weighs the evidence in his *History* (pp. 118–20) and in the first volume of his edition of Euclid [31, pp. 35–8]. What Proclus himself says is as follows; the descriptive headings are by me.

2.7.1. Origin of geometry (65.7)

> Thales, who had travelled to Egypt, was the first to introduce this science into Greece.

Proclus has discussed the origin of geometry in measuring lands after the Nile floods, as contrasted with the origin of arithmetic in the trading and exchange of the Pheonicians. In an article called "Abscissas and Ordinates" [61, pp. 242–3], I dispute the kind of materialistic account given by Proclus. The flooding of the Nile does not make you invent geometry; you invent geometry in order to deal with the flooding of the Nile. It is a question of responsibility: a river has none, but we do.

2.7.2. Bisection of circle (157.11)

> The famous Thales is said to have been the first to demonstrate that the circle is bisected by the diameter. The cause of this bisection is the undeviating course of the straight line through the center; for since it moves through the middle and throughout all parts of its identical movement refrains from swerving to either side, it cuts off equal lengths of the circumference on both sides. If you wish to demonstrate this mathematically, imagine the diameter drawn and one part of the circle fitted upon the other. If it is not equal to the other, it will fall either inside or outside it, and in either case it will follow that a shorter line is equal to a longer . . .

This is in the commentary on Definition XVII. Pappus gives the definition itself as follows.

> A diameter of the circle is a straight line (*εὐθεῖά τις*: Heath "any straight line") drawn through the center and terminated in both directions by the circumference of the circle; and such

> a straight line also bisects the circle.

The Greek in Friedlein's edition agrees with that of Heiberg's edition [30] of Euclid, and Morrow's translation agrees with Heath's [32] except at the point indicated. (Also Heath italicizes "diameter" and has a comma where Morrow has a semicolon.) That the latter part of the "definition" is really a theorem is reason to think that it was not part of Euclid's original text. That Proclus elaborates at such length on a proof suggests that the theorem is not obvious.

It is perhaps odd that a Platonist like Proclus would refer to the "motion" of the diameter through the circle, when in the *Republic* [63, 527A] Socrates has ridiculed those who speak as if geometry were about *doing* things. Geometry is knowledge of something that always *is.* However, Proclus has already addressed this issue, as Seidenberg observes [70, pp. 265–6]. Speusippus (Plato's nephew and successor at the Academy) thought all problems were really theorems; Menaechmus, all theorems were problems. both were right, says Proclus [66, 78.14–22]:

> The school of Speusippus are right because the problems of geometry are of a different sort from those of mechanics, for example, since the latter are concerned with perceptible objects that come to be and undergo all sorts of change. Likewise the followers of Menaechmus are right because the discovery of theorems does not occur without recourse to matter, that is, intelligible matter (*ὕλην τὴν νοητήν*). In going forth into this matter and shaping it, our ideas are plausibly said to resemble acts of production . . .

Heath cites M. Cantor (not G. Cantor!) in saying that Thales's Theorem

> may . . . have been suggested by the appearance of certain figures of circles divided into a number of equal sectors by 2,

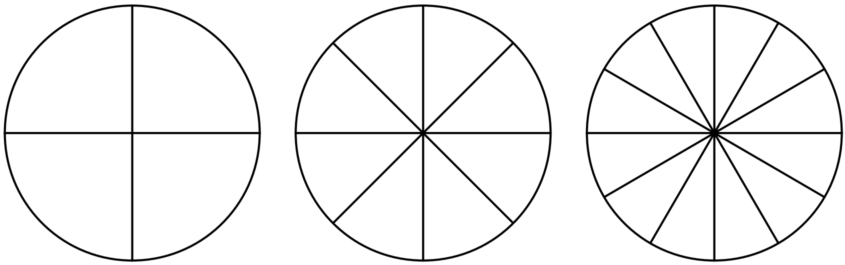


Figure 2.1.: The diameter divides the circle

> 4, or 6 diameters such as are found on Egyptian monuments or represented on vessels brought by Asiatic tributary kings in the time of the eighteenth dynasty.

See Figure 2.1.

If you pointed out to somebody that the sectors of the circle were all equal to one another, I should think the response would be, "So what?" I would expect the same response if you observed that the two halves of the circle made by any diameter were equal. But perhaps not; perhaps there are people who have no conception of comparing two things. The recent book *Sapiens* [39, p. 55] points out how, since the Agricultural Revolution around 10,000 B.C.E., many of us need not know much about the world in order to survive:

> The human collective knows far more today than did the ancient bands. But at the individual level, the ancient foragers were the most knowledgeable and skilful people in history.

Alternatively, perhaps what is remarkable is Thales's recognition, not of the mere equality of the two halves of a circle, but of some kind of "necessity" in their equality. The necessity may have been something along the lines suggested by Proclus. The vertical (that is, head-to-head) sectors created by any two diameters are equal to one another; therefore, by adding up sectors and their opposites in one of the circles in Figure 2.1, we establish the equality of two semicircles.

A sector of a circle resembles an isosceles triangle, considered in the next passage; and the equality of vertical sectors is related to the equality of vertical *angles* in the passage after that.

See page 84 in §4.2 on generalizing the bisection of circles to ellipses.

2.7.3. Isosceles triangles (250.20)

> We are indebted to old Thales for the discovery of this and many other theorems. For he, it is said, was the first to notice and assert (*ἐπιστῆσαι καὶ ἐιπεῖν*) that in every isosceles the angles at the base are equal, though in somewhat archaic fashion he called the equal angles similar (*τὰς ἴσας ὁμοίας*).

This is Euclid's Proposition V. Again there is the question of how obvious the theorem is. According to Heath [42, p. 131],

> It has been suggested that the use of the word 'similar' to describe the equal angles of an isosceles triangle indicates that Thales did not yet conceive of an angle as a magnitude, but as a *figure* having a certain *shape,* a view which would agree closely with the idea of the Egyptian *se-qeṭ*, 'that which makes the nature', in the sense of determining a similar or the same inclination in the faces of pyramids.

Presumably Heath's *se-qeṭ* is Gow's *seqt* in §2.7.5. It does not sound as if Heath has understood that even equality is not sameness in Euclid; see §4.4, page 87. By the definition in the *Elements,*

> A *plane angle* is the inclination (*κλίσις*) to one another of two lines in a plane which meet one another and do not lie in a straight line.

Apparently Heath takes "inclination" here as an abstraction, although it might be understood as an instance of being inclined, and in particular as a figure. According to the Liddell–Scott–Jones lexicon [54], a *κλίσις* can even be a sunset. In his translation of the *Elements,* which predates his *History,* Heath offers to Thales's use of similarity the comparison with

> Arist. *De caelo* IV. 4, 311 b 34 *πρὸς ὁμοίας γωνίας φαίνεται φερόμενον* where *equal* angles are meant.

I should think the key to the present theorem would be that two angles ABC and CBA are equal, that is, congruent; and if $AB = BC$, this additional congruence establishes the congruence of the angles at A and C. This is Pappus's proof, according to Proclus, who gives it just before the quotation above.

Aristotle gave a proof different from Pappus's and Euclid's, but only for the sake of illustrating the syllogism. The relevant passage in the *Prior Analytics* is quoted by Heath [31, p. 253] and Thomas [74, pp. 428–31], but it may be useful to include also the preceding paragraph [7, I.XXIV, $41^{\mathrm{b}}7$–22, pp. 322–5].

> Further, in every syllogism one of the terms must be positive, and universality must be involved. Without universality either there will be no syllogism, or the conclusion will be unrelated to the assumption, or there will be a *petitio principii* (*τὸ ἐξ ἀρχῆς αἰτήσεται*). Suppose that we have to prove that musical enjoyment is commendable. Then if we postulate that enjoyment is commendable, unless 'all' is prefixed to 'enjoyment,' there will be no syllogism. If we postulate that some enjoyment is commendable, then if it is a different enjoyment, there is no reference to the original assumption; and if it is the same, there is a *petitio principii* (*τὸ ἐξ ἀρχῆς λαμβάνει*).

It seems three arguments are contemplated:

1. Enjoying music is commendable, because some enjoyment is commendable.
2. Enjoying music is commendable, because enjoyment of music is commendable.
3. Enjoying music is commendable, because all enjoyment is commendable.

The first is invalid; the second is begging the question; the third assumes even more than what is to be proved, but is nonetheless considered a valid syllogism.

This does not make the syllogism unworthy of study. As "negative evidence" for his notion that Euclid does not employ the "axiomatic method," Seidenberg [70, p. 281–3] notes Euclid's omission of the theorem that circumferences of circles are to one another as the diameters. Euclid omitted the theorem, because he could not prove it. It could be proved only with axioms, such as Archimedes gives in *On the Sphere and Cylinder I* [1, p. 36]:

> /1/ That among lines which have the same limits, the straight ⟨line⟩ is the smallest. /2/ And, among the other lines (if, being in a plane, they have the same limits): that such ⟨lines⟩ are unequal, when they are both concave in the same direction and either one of them is wholly contained by the other and by same straight ⟨line⟩ having the same limits as itself, or some is contained, and some it has ⟨as⟩ common; and the contained is smaller.

Archimedes uses axioms these implicitly in *Measurement of a Circle* [40, pp. 91–3] to show that the circle is equal to the right triangle whose legs are respectively equal to the circumference and the radius. Says Seidenberg,

> EUCLID would have been thunderstruck! It would never have occurred to him that to prove a theorem ("the arc is greater

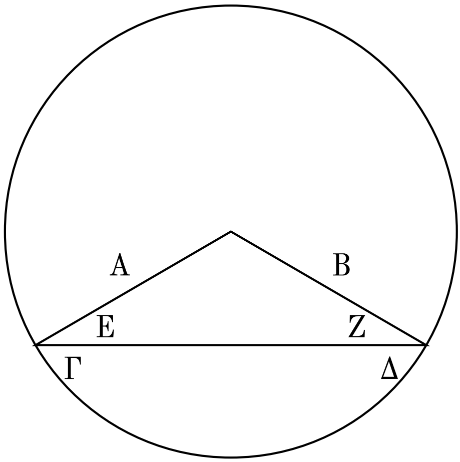


Figure 2.2.: Aristotle's proof of Euclid I.5

> than the chord"), it is all right to generalize it, and then assume the generalization. In fact, though with the Parallel Postulate he may have admitted he was stumped, there is no clear evidence that he thought it was all right to make any geometrical assumption whatever.

Aristotle makes the point about syllogisms with a mathematical example. I revert to Aristotle's letters (the Loeb translation uses Latin letters, the way Heath does). The diagram, apparently not found in the manuscripts, is supposed to be as in Figure 2.2, where Γ, Δ, E, and Z are the angles indicated (the first two having an arc of the circle as a common side), while the angles AΓ and BΔ are the angles made with that arc by the radii A and B respectively.

> The point can be seen more clearly in the case of geometrical theorems. *E.g.,* take the proposition that the angles adjacent to the base of an isosceles triangle are equal. Let the lines A and B be drawn to the centre. Then if you assume that

> $\angle \mathrm{A\Gamma} = \angle \mathrm{B\Delta}$ without postulating generally that the angles of semicircles are equal, and again if you assume that $\angle\Gamma = \angle\Delta$ without also assuming that all angles of the same segment are equal, and further if you assume that when equal angles are subtracted from the whole angles the remaining angle E and Z are equal, unless you assume ⟨the general principle⟩ that when equals are subtracted from equals the remainders are equal, you will be guilty of a *petitio principii.*

The proof is bizarre, because the premises seem less clear than the conclusion. Nonetheless, perhaps Aristotle thought it a good proof, because the combination of a circle and a straight line is simpler than a triangle (the combination of three straight lines). Knowledge of the former kind of configuration perhaps ought to precede that of the latter.

2.7.4. Vertical angles (299.4)

> This theorem, then [namely XV], proves that, when two straight lines cut one another, their vertical angles are equal. It was first discovered by Thales, Eudemus says, but was thought worthy of a scientific demonstration only by the author of the *Elements*.

Proclus observes that Euclid's proof relies on Proposition XIII (that a straight line, stood on another straight line, makes either right angles or angles equal to two right angles) and two axioms (*ἀξιώμασι δυοῖν*, that equals to the same are equal to one another, and remainders are equal when equals are subtracted from equals). A note from Ian Mueller observes that Postulate IV (equality of all right angles) is also used. Seidenberg [70, pp. 270–1] also observes Proclus's failure to mention the fourth postulate and concludes ultimately that this postulate was an interpolation (possibly by Euclid himself).

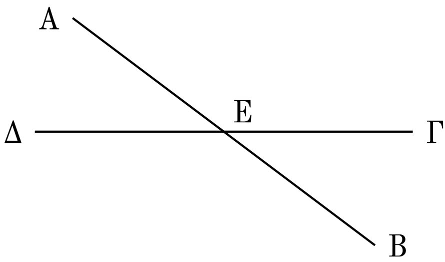


Figure 2.3.: Vertical angles are equal

As I noted above, one might infer the equality of vertical angles from looking at Figure 2.1. Again one might verify it by symmetry. In Figure 2.3, where the straight lines AB and ΓΔ meet at E, and the angle AEΓ is equal to ΓEA, if one demonstrates this equality by flipping the diagram over, one finds $\angle \mathrm{AE\Delta} = \angle \Gamma\mathrm{EB}$. Of course Euclid avoids this kind of proof.

2.7.5. Congruent triangles (352.15)

> Eudemus in his history of geometry attributes the theorem itself [namely ASA and AAS, Proposition XVI] to Thales, saying that the method by which he is reported to have determined the distance of ships at sea shows that he must have used it.

The last quotation from Proclus is number **80** of Kirk *& al.*, who suggest that the method of measuring distances at sea was "similar triangles," and that a "primitive theodolite" could have been used, "two sticks (one as a sight-line, the other as an approximate level-line) pivoting on a nail." But could sufficient accuracy have been achieved with this method? I should think the theodolite would have to be high above the sea, on a hillside; and then we would be back to the question of how to measure this height. Perhaps two observation points along the shore, at a known distance from one another, were used instead.

They do not make them official quotations; but Kirk *&* *al.* mention the accounts of Pliny and Plutarch that constitute DK 11A21. A fuller quotation of Plutarch was in §2.4.1; Kirk *&* *al.* summarize the part of interest:

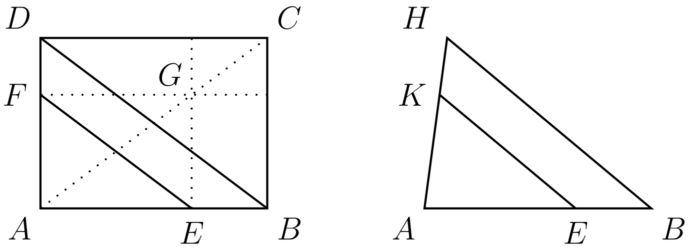
> that the height of a pyramid is related to the length of its shadow exactly as the height of any mensurable vertical object is related to the length of *its* shadow at the same time of day.

According to Kirk *&* *al.*, **80** shows that Thales may have used this more general method, not just the one in **79.** But **80** says nothing about similar triangles.

The last four quotations above from Proclus are found also in Thomas [74, pp. 164–7], who in a note describes Heath's suggested method of measuring the distance of a ship [42, pp. 132–3]. Climb a tower, note the angle of depression of the ship, then find an object on land at the same angle: the object's distance is that of the ship. This obviates any need to know the height of the tower, or to know proportions. Supposedly one of Napoleon's engineers measured the width of a river this way.

Nonetheless, Gow [36, p. 141] suggests that this method is not generally practical, and Thales must have had the more refined method of similar triangles:

> It is hardly credible that, in order to ascertain the distance of the ship, the observer should have thought it necessary to reproduce and measure on land, in the horizontal plane, the enormous triangle which he constructed in imagination in a perpendicular plane over the sea. Such an undertaking would have been so inconvenient and wearisome as to de-



(a) Right triangles (b) Scalene triangles

Figure 2.4.: Similar triangles

> prive Thales' discovery of its practical value. It is therefore probable that Thales knew another geometrical proposition: viz. 'that the sides of equiangular triangles are proportional.' (Euc. VI. 4.)

But here is a subtle point. For the proposed method for measuring distances, Book VI of Euclid is overkill; all one needs is the proposition that, in Figure 2.4a, where $ABCD$ is a rectangle,

$$EF \parallel BD \iff AE \cdot DF = AF \cdot BE.$$

This follows from Proposition 43 of Book I of the *Elements* (if AGC is straight, then the two rectangles on either side of it are equal) and its converse. We may rewrite the equation here in the form

$$AE : AF :: BE : DF,$$

by one possible *definition* of proportion of lengths. Now I think we have the real theorem that Gow goes on to discuss:

> And here no doubt we have the real import of those Egyptian calculations of *seqt,* which Ahmes introduces as exercises in arithmetic. The *seqt* or ratio, between the distance of the ship and the height of the watch-tower is the same as that

> between the corresponding sides of any small but similar triangle. The discovery, therefore, attributed to Thales is probably of Egyptian origin, for it is difficult to see what other use the Egyptians could have made of their *seqt,* when found. It may nevertheless be true that the proposition, Euc. VI. 4, was not known, as now stated, either to the Egyptians or to Thales. It would have been sufficient for their purposes to know, inductively, that the *seqts* of equiangular triangles were the same.

Gow is right that Euclid VI.4 need not have been known. But what he seems to mean is that the Egyptians and Thales need only have had a knack for applying the theorem, without having stepped back to recognize the theorem as such. But from Euclid VI.4 we could conclude that if, in Figure 2.4b, AEB is as in Figure 2.4a, and $AK = AF$, then $HK = DF$ (assuming that indeed $EF \parallel BD$ and $EK \parallel BH$). This is a form of Desargues's Theorem, and it can indeed be proved on the basis of Book I of the *Elements* alone, without the theory of proportion of Book V (which relies on an additional axiom, the so-called Archimedean axiom). But I am not aware that anybody else did the work (or even recognized the possibility and desirability of trying doing it) before David Hilbert [46, §14, pp. 24–9]. Nonetheless, Thales could have recognized the simpler theorem based on Figure 2.4a.

2.8. Diogenes Laertius: The angle in a semicircle

Thomas also quotes Diogenes Laertius, i.24–5:

> Pamphila says that, having learnt geometry from the Egyptians, he [Thales] was the first to inscribe in a circle a right-

> angled triangle, whereupon he sacrificed an ox. Others say it was Pythagoras, among them being Apollodorus the calculator.

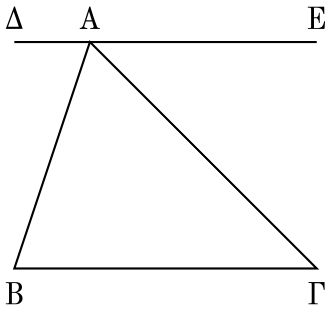
For some reason Kirk *&* *al.* omit this quotation, although their **90** is part of i.24. (They have an index of all sources quoted in the book.) Heath cites the passage in his list of five theorems attributed to Thales [42, pp. 130–1].

Thomas observes that Pamphila was a woman living during the reign of Nero. According to *Wikipedia,* Nero's reign was 54–68; he was last in the Julio-Claudian line Augustus → Tiberius → Caligula → Claudius → Nero. Here Augustus, the first emperor, is the adopted son of Julius Caesar. I have a photo of a stone inscribed ΝΕΡΩΝ from the ruined lighthouse at Patara.

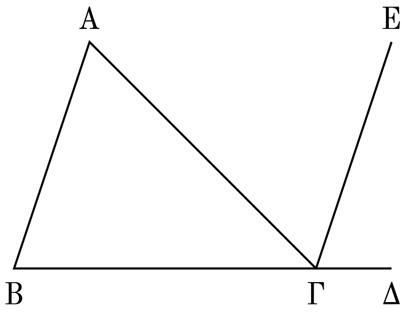
The theorem in question is evidently that the angle in a semicircle is right. This is Euclid's III.31. Euclid's proof is based on his I.32, that the angles in a triangle are equal to two right angles; and according to Proclus at 379.2,

> Eudemus the Peripatetic attributes to the Pythagoreans the discovery of this theorem, that every triangle has internal angles equal to two right angles, and says they demonstrated it as follows

—using Figure 2.5a, as opposed to Euclid's Figure 2.5b (which is needed because Euclid also proves that the exterior angle is equal to the two opposite interior angles). Nonetheless, Heath [42, pp. 136–7] suggests how Thales might have recognized the theorem about semicircles without knowing the general theorem about the angles of a triangle. One might study a rectangle with diagonals as in Figure 2.6a, and observe that the intersection point of the diagonals is equidistant from the four vertices, so those vertices lie on a circle. However, this

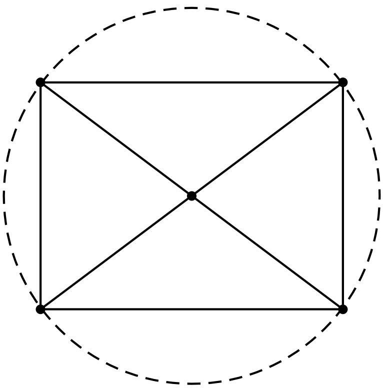


(a) By the Pythagoreans



(b) By Euclid

Figure 2.5.: Euclid's I.32


(a) Right angles in semicircles

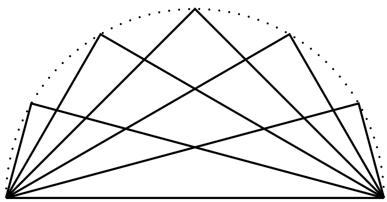

(b) A semicircular locus

Figure 2.6.: Theorems of circles

argument assumes the existence of rectangles, that is, quadrilaterals, each of whose angles is right. They don't exist in non-Euclidean geometry [14]—not that there is any reason to think that Thales recognized this possibility.

Alternatively, given a circle with two diameters drawn as in Figure 2.6a, one may obtain the indicated quadrilateral, which is composed of two vertical pairs of congruent isosceles triangles. So the quadrilateral has equal opposite sides, and all four of its angles are equal. If we grant that these angles must then be right, we are done. But the possibility of covering a floor with square tiles would seem to make this clear.

The suggestiveness of tiles can be seen in Gow [36, p. 144], who passes along a couple of ideas of one Dr Allman. One is that Thales knew inductively that the angles in a triangle are equal to two right angles, "from inspection of Egyptian floors paved with tiles of the form of equilateral triangles, or squares, or hexagons." Since six equilateral triangles meet at a point, and so their angles make up four right angles, each of them has two right angles shared among its angles. Gow mentions similar ideas of Hankel and [M.] Cantor.

Another idea of Dr Allman is that Thales may have found inductively that the *locus* of apices of right triangles whose bases (namely their hypotenuses) are all the same given segment is a semicircle, as in Figure 2.6b. In that case, did Thales not recognize the possibility of *deduction* in mathematics? "Of speculation in this style," says Gow, "there is no end, and there is hardly a single Greek geometer who is not the subject of it."

2.9. Aristotle

Kirk *& al.* observe that "our knowledge of Thales' cosmology depends virtually completely" on their **84** and **85**, from *De Caelo* and the *Metaphysics* respectively. I reserve comment, except that, if Aristotle recognizes four causes, while Thales and the other Ionians know but one, then it will be misleading to think of that one cause as if it were one of Aristotle's four.

2.9.1. *De Caelo*

Quotation **84** is the sentence that I have emboldened, with the two ensuing sentences, in the following passage from Book II of *De Caelo* [4, II.13, pp. 429–30].

> III. There are similar disputes about the *shape* of the earth. Some think it is spherical, others that it is flat and drum-shaped. For evidence they bring the fact that, as the sun rises and sets, the part concealed by the earth shows a straight and not a curved edge, whereas if the earth were spherical the line of section would have to be circular. In this they leave out of account the great distance of the sun from the earth and the great size of the circumference, which, seen from a distance on these apparently small circles appears straight. Such an appearance ought not to make them doubt the circular shape of the earth. But they have another argument. They say that because it is at rest, the earth must necessarily have this shape. For there are many different ways in which the movement or rest of the earth has been conceived.
>
> The difficulty must have occurred to every one. It would indeed be a complacent mind that felt no surprise that, while a little bit of earth, let loose in mid-air moves and will not stay still, and the more there is of it the faster it moves,

the whole earth, free in midair, should show no movement at all. Yet here is this great weight of earth, and it is at rest. And again, from beneath one of these moving fragments of earth, before it falls, take away the earth, and it will continue its downward movement with nothing to stop it. The difficulty then, has naturally passed into a commonplace of philosophy; and one may well wonder that the solutions offered are not seen to involve greater absurdities than the problem itself.

By these considerations some have been led to assert that the earth below us is infinite, saying, with Xenophanes of Colophon, that it has 'pushed its roots to infinity',—in order to save the trouble of seeking for the cause. Hence the sharp rebuke of Empedocles, in the words 'if the deeps of the earth are endless and endless the ample ether—such is the vain tale told by many a tongue, poured from the mouths of those who have seen but little of the whole'. **Others say the earth rests upon water. This, indeed, is the oldest theory that has been preserved, and is attributed to Thales of Miletus.** It was supposed to stay still because it floated like wood and other similar substances, which are so constituted as to rest upon but not upon air. As if the same account had not to be given of the water which carries the earth as of the earth itself! It is not the nature of water, any more than of earth, to stay in mid-air: it must have something to rest upon. Again, as air is lighter than water, so is water than earth: how then can they think that the naturally lighter substance lies below the heavier? Again, if the earth as a whole is capable of floating upon water, that must obviously be the case with any part of it. But observation shows that this is not the case. Any piece of earth goes to the bottom, the quicker the larger it is. These thinkers seem to push their inquiries some way into the problem, but not so far as they might . . .

2.9.2. *Metaphysics*

Quotation **85** of Kirk *& al.* is the second and third paragraphs (with ellipsis of the example involving Socrates) of chapter 3 of Book I of the *Metaphysics* [5, $983^{a}24$, pp. 693–5]. I quote also the first paragraph for its list of the four causes. I introduce the *typographical* enumeration of these causes, and I have taken their Greek names from the Loeb edition [8].

> **3** Evidently we have to acquire knowledge of the original causes (*τῶν ἐξ ἀρχῆς αἰτίων*) (for we say we know each thing only when we think we recognize its first cause), and causes are spoken of in four senses.
>
> [1] In one of these we mean the substance, i.e. the essence (*τὴν οὐσίαν καὶ τὸ τί ἦν εἶναι*) (for the 'why' is reducible finally to the definition, and the ultimate 'why' is a cause and principle);
>
> [2] in another the matter or substratum (*τὴν ὕλην καὶ τὸ ὑποκείμενον*),
>
> [3] in a third the source of the change (*ἡ ἀρχὴ τῆς κινήσεως*), and
>
> [4] in a fourth the cause opposed to this, the purpose and the good (*τὸ οὗ ἕνεκα καὶ τἀγαθόν*) (for this is the end of all generation and change).
>
> We have studied these causes sufficiently in our work on nature [*Physics* II. 3, 7], but yet let us call to our aid those who have attacked the investigation of being and philosophized about reality before us. For obviously they too speak of certain principles and causes; to go over their views, then, will be of profit to the present inquiry, for we shall either find another kind of cause, or be more convinced of the correctness of those which we now maintain.

Briefly, the causes are formal, (1) material, (2) efficient or motor, and (3) final. After this first paragraph, I interject the account of the causes in a different order, and now *with examples,* from the *Physics* [9, B.3, pp. 29–30]; the typographical enumeration is by the translator Apostle:

> In one sense, a "cause" means (1) that from which, as a constituent, something is generated; for example, the bronze is a cause of the statue, and the silver, of the cup, and the genera of these [are also causes].
>
> In another, it means (2) the form or the pattern, this being the formula or the essence, and also the genera of this; for example, in the case of the octave, the ratio 2 : 1, and, in general, a number and the parts in the formula.
>
> In another, it means (3) that from which change or coming to rest first begins; for example, the adviser is a cause, and the father is the cause of the baby, and, in general, that which acts is a cause of that which is acted upon, and that which brings about a change is a cause of that which is being changed.
>
> Finally, it means (4) the end, and this is the final cause [that for the sake of which]; for example, walking is for the sake of health. Why does he walk? We answer, "In order to be healthy"; and having spoken thus, we think that we have given the cause. And those things which, after that which started the motion, lie between the beginning and the end, such as reducing weight or purging or drugs or instruments in the case of health, all of them are for the sake of the end; and they differ in this, that some of them are operations while others are instruments.
>
> The term "cause", then, has about so many senses. And since they [the causes] are spoken of in so many ways, there may be many nonaccidental causes of the same thing; for example, in the case of a statue, not with respect to something else but qua a statue, both the art of sculpture and the bronze are causes of it, though not in the same manner; but

the bronze as matter and the art as source of motion. There may be also causes of each other; for example, exercise is a cause of good physical condition, and good physical condition is a cause of exercise, although not in the same manner, but good physical condition as an end, while exercise as a principle of motion . . .

Now we continue where we left off, with the next few paragraphs from the *Metaphysics,* I.3.

> **Of the first philosophers, then, most thought the principles which were of the nature of matter were the only principles of all things** (Τῶν δὴ *πρώτων φιλοσοφησάντων οἱ πλεῖστοι τὰς ἐν ὕλης εἴδει μόνας ᾠήθεσαν ἀρχὰς εἶναι πάντων*). That of which all things that are consist, the first from which they come to be, the last into which they are resolved (the substance remaining, but changing in its modifications), this they say is the element and this the principle of things[1] (*ἐξ οὗ γὰρ ἔστιν ἅπαντα τὰ ὄντα, καὶ ἐξ οὗ γίγνεται πρώτου καὶ εἰς ὃ φθείρεται τελευταῖον, τῆς μὲν οὐσίας ὑπομενούσης, τοῖς δὲ πάθεσι μεταβαλλούσης, τοῦτο στοιχεῖον καὶ ταύτην ἀρχήν φασιν εἶναι τῶν ὄντων*), and therefore they think nothing is either generated or destroyed, since this sort of entity is always conserved, as we say Socrates neither comes to be absolutely when he comes to be beautiful or musical, nor ceases to be when loses these characteristics, because the substratum, Socrates himself, remains. Just so they say nothing else comes to be or ceases to be; for there must be some entity—either one or more than one—from which all other things come to be, it being conserved.
>
> Yet they do not all agree as to the number and the nature

[1]Because of the use of gender, I wonder if a "respectively" is implied, as if the meaning is, "That of which all things consist they call the element; that from which they come to be and into which they are resolved, the principle."

of these principles. **Thales, the founder of this type of philosophy, says the principle is water (for which reason he declared that the earth rests on water),** getting the notion perhaps from seeing that the nutriment of all things is moist, and that heat itself is generated from the moist and kept alive by it (and that from which they come to be is a principle of all things). He got his notion from this fact, and from the fact that the seeds (*τὰ σπέρματα*) of all things have a moist nature, and that water is the origin of the nature of moist things.

Some think that even the ancients who lived long before the present generation, and first framed accounts of the gods, had a similar view of nature; for they made Ocean and Tethys the parents of creation, and described the oath of the gods as being by water, to which they give the name of Styx; for what is oldest is most honourable, and the most honourable thing is that by which one swears. **It may perhaps be uncertain whether this opinion about nature is primitive and ancient, but Thales at any rate is said to have declared himself thus about the first cause** (*περὶ τῆς πρώτης αἰτίας*). Hippo [of Samos] no one would think fit to include among these thinkers, because of the paltriness of his thought.

Anaximenes and Diogenes [of Apollonia, contemporary with Hippo, latter half of 5th c.] make air prior to water, and the most primary of the simple bodies, while Hippasus of Metapontium and Heraclitus of Ephesus say this of fire, and Empedocles says it of the four elements (adding a fourth—earth—to those which have been named); for these, he says, always remain and do not come to be, except that they come to be more or fewer, being aggregated into one and segregated out of one.

Anaxagoras of Clazomenae, who, though older than Empedocles, was later in his philosophical activity, says the principles are infinite in number; for he says almost all the things that are made of parts like themselves, in the manner of water or fire, are generated and destroyed in this way, only by aggregation and segregation, and are not in any other sense generated or destroyed, but remain eternally.

2.9.3. *De Anima*

Finally, the three quotations **89, 86,** and **91** of Kirk *& al.* that are from *De Anima* are the following [3, 6].

The magnet has a soul ($405^{a}19$)

I start earlier, at $404^{b}30$.

As to the nature and number of the first principles opinions differ. The difference is greatest between those who regard them as corporeal and those who regard them as incorporeal, and from both dissent those who make a blend and draw their principles from both sources. The number of principles is also in dispute; some admit one only, others assert several. There is a consequent diversity in their several accounts of soul; they assume, naturally enough, that what is in its own nature originative of movement must be among what is primordial. That has led some to regard it as fire, for fire is the subtlest of the elements and nearest to incorporeality; further, in the most primary sense, fire both is moved and originates movement in all the others.

Democritus has expressed himself more ingeniously than the rest on the grounds for ascribing each of these two characters to soul; soul and mind are, he says, one and the same thing, and this thing must be one of the primary and indivisible bodies, and its power of originating movement must be due to its fineness of grain and the shape of its atoms; he says that of all the shapes the spherical is the most mobile, and that this is the shape of the particles of fire and mind.

Anaxagoras, as we said above, seems to distinguish between soul and mind, but in practice he treats them as a single substance, except that it is mind that he specially posits as the principle of all things; at any rate what he says is that mind alone of all that is is simple, unmixed, and pure. He assigns both characteristics, knowing and origination of movement, to the same principle, when he says that it was mind that set the whole in movement.

Thales, too, to judge from what is recorded about him, seems to have held soul to be a motive force, since he said that the magnet has a soul in it because it moves the iron.

Hippo on the soul as water (405^b1)

Kirk *& al.* give the following quotation in a footnote to their commentary on the *Metaphysics* quotation (**85**), speculating that Hippo might be the source of Aristotle's explanation of why Thales thought water was the principle.

> Of more superficial writers, some, e.g. Hippo, have pronounced it [the soul] to be water; they seem to have argued from the fact that the seed (*γονή*) of all animals is fluid, for Hippo tries to refute those who say that the soul is blood, on the ground that the seed, which is the primordial soul (*τὴν πρώτην ψυχήν*), is not blood.

All things are full of gods (411^a7)

I start at 411^a2.

> If we must construct the soul out of the elements, there is no necessity to suppose that all the elements enter into

its construction; one element in each pair of contraries will suffice to enable it to know both that element itself and its contrary. By means of the straight line we know both itself and the curved—the carpenter's rule enables us to test both—but what is curved does not enable us to distinguish either itself or the straight.

Certain thinkers say that soul is intermingled in the whole universe, and it is perhaps for that reason that Thales came to the opinion that all things are full of gods (*πάντα πλήρη θεῶν εἶναι*). This presents some difficulties: Why does the soul when it resides in air or fire not form an animal, while it does so when it resides in mixtures of the elements, and that although it is held to be of higher quality when contained in the former?

3. Interpretations

Several modern books in my possession comment on the Ionian philosophers. The books may have a better understanding of Thales than Aristotle does, though we remain indebted to Aristotle (and the scribes who copied him over the centuries) for giving us any idea of what Thales thought in the first place.

3.1. Kant

First I observe, from the Preface to the B Edition of the *Critique of Pure Reason,* Kant's attribution to somebody, who may well be Thales, of the creation of mathematics as a science [50, B x–xi, p. 107].

> **Mathematics** has, from the earliest times to which the history of human reason reaches, in that admirable people the Greeks, traveled the secure path of a science. Yet it must not be thought that it was as easy for it as for logic—in which reason has to do only with itself—to find that royal path, or rather itself to open it up; rather, I believe that mathematics was left groping about for a long time (chiefly among the Egyptians), and that its transformation is to be ascribed to a **revolution,** brought about by the happy inspiration of a single man in an attempt from which the road to be taken onward could no longer be missed, and the secure course of a science was entered on and prescribed for all time and to an infinite extent. The history of this revolution in the way of

> thinking—which was for more important than the discovery of the way around the famous Cape—and of the lucky one who brought it about, has not been preserved for us. But the legend handed down to us by Diogenes Laertius—who names the reputed inventor of the smallest elements of geometrical demonstration, even of those that, according to common judgment, stand in no need of proof—proves that the memory of the alteration wrought by the discovery of this new path in its earliest footsteps must have seemed exceedingly important to mathematicians, and was thereby rendered unforgettable. A new light broke upon the person who demonstrated the isosceles triangle (whether he was called "Thales" or had some other name). For he found that what he had to do was not to trace what he saw in this figure, or even trace its mere concept, and read off, as it were, from the properties of the figure; but rather that he had to produce the latter from what he himself thought into the object and presented (through construction) according to *a priori* concepts, and that in order to know something securely *a priori* he had to ascribe to the thing nothing except what followed necessarily from what he himself had put into it in accordance with its concept.

As quoted from the *New Leviation* in §2.5 (page 34), Collingwood takes up the theme that, in a civilization, once one person makes a discovery, we can all benefit from it. However, the title of *The Forgotten Revolution* (used in §4.2, page 81) alludes to Russo's theme: a discovery may not be passed along, if people lose the ability to recognize its value. Such was the fate of, for example, the last of the eight books of Apollonius on conic sections; Books V–VII survive in Arabic, but only Books I—IV survive in the original Greek, presumably because the later books were found too difficult by anybody who could afford to have copies made [69, p. 8].

3.2. Collingwood

The Idea of Nature [17] examines the absolute presuppositions about nature made in (1) ancient Greece, (2) Renaissance Europe, and (3) modern times. Collingwood is providing an example of the work of the metaphysician as described in *An Essay on Metaphysics* [20]. After an Introduction, *The Idea of Nature* starts in with the Ionians:

> According to Aristotle, the characteristic of this Ionian cosmology is the fact that whenever its devotees ask the question: **'What is nature?'** they at once convert it **into the question: 'What are things made of?'** or 'What is the original, unchanging substance which underlies all the changes of the natural world with which we are acquainted?'

It appears Collingwood takes Aristotle seriously, even though, in a footnote, he acknowledges a warning about this; his response for now is that the critic himself follows Aristotle tacitly:

> Monsieur E. Brehier (*Histoire de la Philosophie,* Paris, 1928, vol. i, p. 42) says that the question 'What are things made of?' is not Thales' question but Aristotle's question. There is certainly force in his warning that **our traditional view of the Ionian physicists through the spectacles of Aristotle places us in danger of ascribing exaggerated importance in the minds of these men to what may in fact have been little more than *obiter dicta,*** and thus projecting fourth century problems back into the sixth century or even the late seventh. Yet Monsieur Brehier himself says 'Le phénomène fondamental dans cette physique milésienne est bien l'évaporation de l'eau de la mer sous I'influence de la chaleur' (p. 44). In other words, Monsieur Brehier in spite of his own warning continues to accept Aristotle's view that the fundamental concept of Ionian physics was the con-

cept of transformation.

Meanwhile, Collingwood continues in the main text:

> People who could ask this question must have already settled in their minds a large number of preliminary points . . . I will mention three of them.
>
> 1. *That there are 'natural' things . . .*
>
> 2. *That 'natural' things constitute a single 'world of nature' . . .*
>
> 3. *That* ***what is common to all 'natural' things is their being made of a single 'substance' or material.*** This was the special or peculiar presupposition of Ionian physics; and the school of Miletus may be regarded as a group of thinkers who made it their special business to take this as their 'working hypothesis' and see what could be made of it: asking in particular the question: 'That being so, what can we say about this single substance?' They did not consciously treat it as a 'working hypothesis': it cannot be doubted that **they accepted it as an absolute and unquestioned presupposition of all their thinking;** but the historian of thought, looking back on their achievement, cannot fail to see that **what they really did was to test this idea of a single universal substance and to find it wanting.**

I would take issue with asserting at the same time that oneself knows what another person is "really" doing, but that the other person is capable of recognizing it. Freud speaks this way, saying somewhere that the reason for a neurosis cannot just be told to the person suffering it. In case of neurosis though, there is a chance that the patient will discover the reason for it through analysis. We can put Thales on the couch only in our own imaginations.

In *An Essay on Metaphysics* [20, p. 40], Collingwood describes "metaphysical analysis," which is "the analysis which detects absolute presuppositions."

> Such analysis [says Collingwood on page 43 of the *Essay*] may in certain cases proceed in the following manner. If the enquirer can find a person to experiment on who is well trained in a certain kind of scientific work, intelligent and earnest in his devotion to it, and unaccustomed to metaphysics, let him probe into various presuppositions that his 'subject' has been taught to make in the course of scientific education, and invite him to justify each or alternatively to abandon it . . . when an absolute presupposition is touched, the invitation will be rejected, even with a certain degree of violence . . .
>
> This is a precarious method, because the qualifications it demands in the subject are too delicate . . . Perhaps there was a kind of justice in the allegation that Socrates, the great master of this method, 'corrupted the young men' . . . The only altogether satisfactory method is for the analyst to experiment on himself . . .

It is hard to get somebody to recognize his absolute presuppositions; but it is not impossible. Could Thales have been induced to recognize the absolute presuppositions that Collingwood attributes to him? A passage in *The Principles of History* [22, p. 30] suggests that Collingwood may not think this is important:

> Confronted with a ready-made statement about the subject he is studying, the scientific historian never asks himself: 'Is this statement true or false?', in other words 'Shall I incorporate it in my history of that subject or not?' The question he asks himself is: 'What does this statement mean?' And this is not equivalent to the question 'What did the person who made it mean by it?', although that is doubtless a

> question that the historian must ask, and must be able to answer. It is equivalent, rather, to the question 'What light is thrown on the subject in which I am interested by the fact that this person made this statement, meaning by it what he did mean?'

Thus, according the last quotation from *The Idea of Nature,* Thales meant that what is common to all natural things is their being made of water; but this ultimately means to us is that what is common to all natural things cannot be a "substance" at all.

Concerning Thales, Collingwood goes on to say, "He held, as everyone knows, that the universal substance out of which things are made is water." This gives us two questions:

1. Why water?
2. How does "a thing made of water, such as a stone or a fish," differ from the water itself?

> On the second question we have no light at all. On the first, Aristotle himself has no information, but he has put forward two suggestions which are admittedly guesses. The first is that moisture is necessary for the nourishment of every organism; the second, that every animal's life begins in seminal fluid.
>
> The point to be noticed here is not what Aristotle says but what it presupposes, namely that **Thales conceived the world of nature as an organism: in fact, as an animal.** This is confirmed by the fragments which have come down to us of Thales' own utterances. According to these fragments, Thales regarded the world (the earth *plus* the heavens, that is to say; what later Greek thinkers called *κόσμος*, but the Milesians called *οὐρανός*,) as something 'ensouled', *ἔμψυχον*, a living organism or animal, within which are lesser organisms having souls of their own . . . **he may**

possibly have conceived the earth as grazing, so to speak, on the water in which it floats, thus repairing its own tissues and the tissues of everything in it by taking in water from this ocean and transforming it, by processes akin to respiration and digestion, into the various parts of its own body. We are told, moreover, that **he described the world as *ποίημα θεοῦ*, something made by God.** That is to say, the vital processes of this cosmic organism were not conceived by him as self-existent or eternal (for he said that God is 'older' than the world) but as depending for their existence on an agency prior to them and transcending them.

It is evident from these scanty records that the ideas of Thales were enormously remote from the Renaissance conception of the natural world as a cosmic machine made by a divine engineer in order to serve his purposes. He regarded it as a cosmic animal whose movements, therefore, served purposes of its own. **This animal lived in the medium out of which it was made, as a cow lives in a meadow. But now the question arose, How did the cow get there?** What transformed the undifferentiated water into that mass of differentiated and ensouled water which we call the world? Here the analogy between the world and a cow breaks down. The cosmic cow did not begin its life as a calf. The life of the world-animal does not include anything analogous to reproduction. **The world was not born, it was made; made by the only maker that dare frame its fearful symmetry: God.**

But what kind of a making was this? It was very unlike that making which Renaissance cosmology attributed to the 'great architect of the universe'. For Renaissance thought, as that phrase indicates, the creative activity of God in its relation to the world of nature is in all points except one a scaled-up version of the activity by which a man builds

> a house or a machine; the one exception being that God is an architect or engineer who has no need of materials but can make His world out of nothing. If the divine activity of which Thales spoke in his phrase *ποίημα θεοῦ* is a scaled-up version of any human activity, this human activity is not the activity of an architect or engineer but the activity of a magician. God, in the cosmology of Thales, makes a cosmic animal out of water **as magically as Aaron made a snake out of a walking-stick, or as the Arunta in their *inchitiuma* ceremonies make a supply of emus or witchetty grubs.**

This ends Collingwood's specific treatment of Thales. He continues with Anaximander and Anaximenes, then considers "Limits of Ionian natural science" and "Meaning of the word 'nature.' "

3.3. Frankfort and Frankfort

I bought *Before Philosophy* [35] in a used bookshop somewhere; possibly this was in Annapolis when I was a freshman at St John's, but I do not recall clearly.

In the Conclusion of the book, the Frankfort couple write movingly of the Ionians. I note two themes:

1. Greek thought is mathematical in the sense of being elaborated by deductive reasoning. See the end of my selection on this. (The Frankforts do not seem to mention mathematics as such, and it is not in their index.)
2. Thales's water is not to be understood like the elements on the Periodic Table; but one should recall that the land is not always green, but becomes so when rain comes; and Homer [47, 14.201, 245–6] refers to

> Oceanus, from whom the gods are sprung

and

> the streams of the river Oceanus from whom they [the gods] all are sprung

Here now the Frankforts, from their pages 248–61.

> In the sixth century B.C. the Greeks, in their great cities on the coast of Asia Minor, were in touch with all the leading centres of the civilized world: Egypt and Phoenicia; Lydia, Persia, and Babylon. There can be no doubt that this contact played some part in the meteoric development of Greek culture . . .
>
> And yet **Hesiod was without oriental precedent in one respect: the gods and the universe were described by him as a a matter of private interest.** Such freedom was unheard of in the Near East, except among the Hebrews, where Amos, for instance, was a herdsman . . .
>
> The same freedom, the same unconcern as regards special function and hierarchy, is characteristic for the Ionian philosophers who lived a century or more after Hesiod. Thales seems to have been an engineer and statesman; Anaximander, a map-maker . . .
>
> . . . Like Hesiod, the Ionian philosophers gave their attention to the problem of origins; but for them it assumed an entirely new character. The origin, the ἀρχή, which they sought was not understood in the terms of myth . . .
>
> Yet the doctrines of **the early Greek philosophers** are not couched in the language of detached and systematic reflection. Their sayings sound rather like inspired oracles. And no wonder, for these men **proceeded, with preposterous boldness, on an entirely unproved assumption.** They held that **the universe is an intelligible whole . . .**
>
> The speculative courage of the Ionians is often overlooked.

Their teachings were, in fact, predestined to to be misunderstood by modern—or rather, nineteenth-century—scholars. When Thales proclaims water to be the first cause, or Anaximenes air; when Anaximander speaks of the 'boundless', and Heraclitus of fire; when, moreover, Democritus' theory of atoms can be considered the outcome of these earlier speculations; then we need not be astonished that **commentators in a positivistic age unwittingly read familiar connotations into the quasi-materialist doctrines of the Ionians** and regard these earliest philosophers as the first scientists. No bias could more insidiously disfigure the greatness of the Ionian achievement. **The materialist interpretation of their teachings takes for granted what was to be discovered only as a result of the labours of these ancient thinkers—the distinction between the objective and the subjective.** And only on the basis of this distinction is scientific thought possible.

In actual fact the Ionians moved in a curious borderland. They forefelt the possibility of establishing an intelligible coherence in the phenomenal world; yet they were still under the spell of an undissolved relationship between man and nature . . .

. . . Anaximenes recognized in air something variable enough to make it seem possible to interpret the most diverse phenomena as its manifestations. Thales had preferred water, but he, too, did not consider his first cause merely as a neutral, colourless liquid. We must remember that **seeds and bulbs and the eggs of insects lie lifeless in the rich soil of Eastern Mediterranean lands until the rains come**—remember, also, the preponderant role of watery substances in the processes of conception and birth in the animal kingdom. It is possible that the ancient oriental view of water as a fertilizing agent had retained its validity

> for Thales. It is equally possible that he endorsed the oriental conception of a primeval ocean from which all life came forth. Homer, as we have seen, called Okeanos the origin of gods and men . . .
>
> . . . In the first place, early Greek philosophy (in Cornford's words) 'ignored with astonishing boldness the prescriptive sanctities of religious representation' [*Cambridge Ancient History,* IV, 532]. Its second characteristic is a passionate consistency. Once a theory is adopted, it is followed up to its ultimate conclusion irrespective of conflicts with observed facts or probabilities. Both of these characteristics indicate an implicit recognition of the autonomy of thought; they also emphasize the intermediate position of early Greek philosophy . . . Its disregard for the data of experience in its pursuit of consistency distinguishes it from later thought . . .
>
> . . . With conviction they propounded theories which resulted from intuitive insight and which were elaborated by deductive reasoning . . .

3.4. Guthrie

W. K. C. Guthrie, *The Greek Philosophers* [38], was used in a required course of ancient Greek history, in the year *after* I took the course as a freshman in high school. When I expressed interest, the teacher gave me a copy of Guthrie's book. When I had taken the course, we had read some of Cornford's version of the *Republic* (perhaps only the passages on the Divided Line and the Cave). Guthrie suggests in a note on his page 2 that the job of his essay "has been done, as well as it is ever likely to be, by F. M. Cornford in *Before and After Socrates* (Cambridge University Press, 1932)." And yet, at a Friday-night lecture at St John's College, a speaker invited us to burn our copies of Cornford's *Republic,* if we had them, because Cornford distorted the harmonious ratios that the speaker thought Plato had built into the timing of the dialogue.

For Guthrie (p. 23), the Milesian School

> [1] looked for something permanent, persisting through the chaos of apparent change; and they [2] thought that they would find it by asking the question: 'What is the world made of?'

I would question this two-part analysis. As Guthrie notes, one could alternatively suppose "that the permanent and comprehensible element lies in its structure or form" (p. 25). But I do not know any reason to think that the Ionians had Aristotle's distinction between form and matter. It's not as if the Ionians were faced with a choice of one or the other.

Thales said the world was made of water. Why? "The explanation which occurs most readily to modern scholars" (p. 25) is that water is seen to exist in three phases: solid, liquid, and gas. If this is what modern scholars tend to think, it only brings us back to the original question. What persists when ice melts and water boils?

After considering Anaximander and Anaximenes, Guthrie says (pp. 31–2, first ellipsis in original),

> As Cornford put it, 'If we would understand the sixth-century philosophers, we must disabuse our minds of the atomistic conception of dead matter in mechanical motion and of the . . . dualism of matter and mind.' Aristotle, who was already criticizing the Ionians for (as it appeared to him) 'lazily shelving' the question of the motive cause, remarks in one place, without comment, that none of them made earth the primary substance. There was surely a good reason for this. They wanted a substance which would *explain its own movement,* as in those early days it was still possible to

> imagine it doing. One thought of the ceaseless tossing of the sea, another of the rushing of the wind . . .

Presently Guthrie quotes the end of passage **85** of Kirk *& al.*, that is, the end of the paragraph ending on page 63 above, as having more to be said for it "than modern commentators are inclined to allow."

4. Proof

Here are some notes concerning the origin of mathematical proof as we know it.

4.1. Plato

Plato's dialogues provide examples of proofs of assertions not normally considered mathematical. For example, in Book X of the *Republic,* Socrates offers a deductive proof of the immortality of the soul as follows.

1. Everything has its own badness, evil, disease: the eyes have ophthalmia; grain, mildew.
2. Nothing is destroyed, except by its own badness.
3. The badness of the soul is injustice, ignorance, &c.
4. These do not kill the soul.
5. Therefore the soul cannot die.

The whole argument is in Appendix C. According to Diogenes [28, I.24],

> some, including Choerilus the poet, declare that he [Thales] was the first to maintain the immortality of the soul.

Do we find arguments for such propositions to be valid and worth making, if we think proofs of *mathematical* assertions are worth making?

4.2. Autolycus and Aristoxenus

Autolycus and Aristoxenus wrote astronomy and music, respectively, around the time of Euclid, possibly earlier, and in somewhat of Euclid's style.

In *The Forgotten Revolution* [69, pp. 48–9], Lucio Russo argues that even if, as is said, the results presented in the *Elements* were known before Euclid,

> the main feature of Euclid's work is not the set of results presented, but the way in which these results connect together, forming infinitely extensible "networks" of theorems, drawn out from a small number of distinguished statements. To judge the originality of the *Elements*, therefore, one must ask whether a similar structure (without which one cannot extend the theory by doing "exercises": that is the whole point!) had been achieved prior to Euclid.

As Russo has explained on his page 17 (though without explicit mention of students), in a deductive science, one can assign the exercise of *proving* a particular theorem. This is possible, because there is universal agreement on what constitutes a proof, once the foundations are laid down.

Russo thinks Euclid must have been the first to lay down such foundations. There are earlier proofs, but none based on such foundations as Euclid's postulates. However, Russo's search for evidence does not go explicitly beyond Plato and Aristotle, as the continuation of the last quotation shows. Here the Plato reference is to *Republic* VI, 510C; the Aristotle, to *Analytica posteriora* I.X, 7a[a]40; and the common notion from Euclid was used in the example from the *Prior Analytics* (page 50).

> In the surviving fragments on pre-Euclidean mathematics there is no evidence for sets of postulates similar to Euclid's.

> The works of Plato and Aristotle, moreover, offer an explicit description of what the "principles" accepted by mathematicians as the initial assumptions of their science were like at the time. Plato writes that "those who work with geometry, arithmetic, and the like lay as 'hypotheses' evenness and oddness, figures, the three kinds of angles and similar things." Aristotle, in a passage where he discusses the role of principles in the deductive sciences, makes a distinction between the principles common to all sciences and those particular to each. As an example of the first type he mentions the assertion "Subtract equals from equals and equals remain", which appears in the *Elements* exactly as one of the "common notions". Immediately before that he had written: "Particular [principles] are 'The line is such-and-such', and likewise for straightness."
>
> There is an obvious difference between the type of "geometric principles" exemplified by Plato and Aristotle, which surely could not serve as the basis for proving theorems, and the postulates contained in the Elements.
>
> As to the premises actually used in the demonstration of geometric theorems, several passages from Plato and Aristotle attest to a deductive method much more fluid in the choice of initial assumptions than that transmitted by the *Elements* and later works.
>
> The logical unity of the *Elements*, or of a large portion of it, is clearly not due to chance; it is the result of conscious work on the part of the same mathematician to whom we owe the postulates. There is no reason to suppose that this unity is not an innovation due to Euclid, and a very important one at that.

Autolycus is not in Russo's index (nor does he come up in a search of the `djvu` file from which I have cut and pasted the quotations above). His work [10, 58] is in the format that Fowler calls "*protasis*-style" in *The Mathematics of Plato's Academy* [34, p. 386–7]. In this style, statements (*protases*) are followed by their proofs, with no other commentary. We might also call this the "theorem–proof" style. Fowler takes up the question of whether mathematics before Euclid "was striving towards the content and style of Euclid's *Elements.*" Taking for the *Elements* the conventional date of 300 B.C.E., Fowler says that, before then,

> We shall find that the *only* evidence for *protasis*-style in this period will be in Aristotle, and most of that . . . will not be in mathematics but in logic, in his *Prior Analytics.* Outside Aristotle, the evidence will be in music, with Aristoxenus' *Elements of Harmony,* Book III; or in contexts which may not actually be pre-Euclidean, like Autolycus, *On Risings and Settings and On the Heavenly Spheres.*

Concerning Aristoxenus, Autolycus, and Euclid, Fowler says on his pages 392–3:

> There are even problems with all of these authors: the name of *Elements of Harmonics* might have been given to Aristoxenus' treatise after the tradition of naming books *Elements of* . . . had been established; the evidence that Autolycus predates Euclid's *Elements* is no more convincing than the other way round; and we know nothing of the composition—place, date, and author or authors—of Euclid's *Elements.* And I do not know whether the use in mathematics of *protasis*-style could be an importation of its use in logic by Aristotle, and know of no discussion of this.

Perhaps we cannot say categorically that Euclid is the originator of *protasis*-style for mathematics; but this style did arise around when Euclid himself (whoever he was) was working, and nobody else used postulates as he did.

Thomas [74, p. 490, n. *a*] thinks Euclid's *Phaenomena* is based on Autolycus, though he does not quote him; he quotes Euclid here only to the extent of:

> If a cone or cylinder be cut by a plane not parallel to the base, the resulting section is a section of an acute-angled cone which is similar to a shield.

I note by the way that this theorem allows a generalization of Thales's supposed theorem that the circle is bisected by the diameter. If an ellipse is obtained by cutting a cylinder obliquely, it still has a center, and a straight line through this center bisects the ellipse; but this theorem may not be so obvious as Thales's theorem, at least if the the line through the center is itself oblique: for in this case that line is not an axis of symmetry.

We obtain the ellipse also as a section of a cone; but here the symmetry of the ellipse is not obvious at all. Much less is it clear that the same curve could be obtained from a cylinder. Thus the theorem about the circle might be seen as opening the way to several interesting theorems.

Fowler discounts Heath's argument [42, pp. 348–53] that Euclid's *Phaenomena* is cribbed from Autolycus: the similarities could mean the cribbing was the other way. Moreover, we do not know whether the *Phaenomena* was written before the *Elements,* or even whether it is by the same person.

Nonetheless, in *The Shaping of Deduction in Greek Mathematics* [59, p. 275], Reviel Netz says,

> The solid starting-point for Euclidean-style geometry is neither Euclid nor Autolycus, but Aristotle.

Consulting Barker, *Greek Musical Writings* [11, p. 170], I see that Book III of Aristoxenus is indeed in *protasis*-style, the first proposition being, "Successive tetrachords are either conjunct or disjunct." Barker explains the style himself in a note:

> The third book is quite unlike the others. It consists of a set of theorems deriving propositions from principles already adopted. The theorems are what Aristoxenus, following Aristotle, calls *apodeixeis,* 'demonstrations', and are thought of as explaining why the propositions are true, as well as proving them (see the introductions to chapter 3 and to this chapter). Each proposition is first stated, then demonstrated (I follow Macran in italicising the initial statements, which I have also numbered, in order to bring out this pattern; it parallels that of Euclid in 8 *Sect. Can.* and in the *Elements of Geometry*).

There is no preamble to Book III of Aristoxenus: he starts right in with propositions. Autolycus begins *On the Heavenly Spheres* with two definitions, the second bracketed by Mogenet, of

1) *ὁμαλῶς φέρεσθαι* "to be borne equably" (said of points) and
2) *ἄξων σφαίρας* "axis of a sphere."

The LSJ [54] does not cite Autolycus for the meaning of the first, although he is on the list of sources for the lexicon.

For my awareness of Autolycus in the first place, I thank Ayşe Berkman for having called to my attention an article [12] in the "Science Technology" (Bilim Teknoloji) supplement of *Cumhuriyet* newspaper, concerning the publication in Turkish of what is supposed to be the oldest book of science, namely that of Autolycus of Pitane.

The *protasis*-style may not be original to Euclid; but the postulates—the very idea of using such postulates—would seem to be original, or at least there is no evidence otherwise.

4.3. Hypsicles

Euclid makes the postulate that all right angles are equal to one another. It should be obvious to everybody that they *are* equal. At least it is tacitly accepted by anybody who uses a set square.

On the other hand, would everybody give Euclid's definition? It is one of the few now bound with the *Elements* that are needed:

> When a straight line set up on a straight line makes the adjacent angles equal to one another, each of the angles is *right* (ὀρθή), and the straight line standing on the other is called a *perpendicular* (κάθετος) to that on which it stands.

One could define a right angle to be any of the angles in an equiangular quadrilateral; but then a postulate that the angles in any two equiangular quadrilaterals were equal would yield the Parallel Postulate.

Some people may prove that all right angles are equal by observing that every right angle measures 90 degrees; but this begs the question of whether all degrees are the same. Inspired by the thought of mountains, one might conceive of the surface of every right cone (bounded by the circumference of its base) as a circle. If a degree is a 360th of the "way around," then different cones will yield different degrees.

In any case, the first division of the circle into 360 degrees by a writer in Greek—even the first indication of any influence of Babylonian sexagesimal arithmetic—is said to be by Hypsicles. One source here is Fowler, *The Mathematics of Plato's Academy* [34, p. 219, n. 52], who observes (pp. 223, 83) that Hypsicles flourished around 150 B.C.E. and wrote the so-called Book XIV of the *Elements.* For Fowler it is important that Babylonian arithmetic did not appear in Greek mathematics till late (p. 399).

4.4. Equality

Equality in Euclid is not sameness, but congruence, as discussed in my paper "Abscissas and Ordinates" [61, pp. 238–40].

Neither is equality sameness in ordinary life. Here is Article 7 of the Universal Declaration of Human Rights:[1]

> All are equal before the law and are entitled without any discrimination to equal protection of the law. All are entitled to equal protection against any discrimination in violation of this Declaration and against any incitement to such discrimination.

In Turkish, this is Madde 7 of the İnsan hakları evrensel beyannamesi:[2]

> Kanun önünde herkes eşittir ve farksız olarak kanunun eşit korumasından istifade hakkını haizdir. Herkesin işbu Beyannameye aykırı her türlü ayırdedici mualeleye karşı ve böyle bir ayırdedici muamele için yapılacak her türlü kışkırtmaya karşı eşit korunma hakkı vardır.

The notion of equality before the law is traced to the Funeral Oration of Pericles in Athens, 431/0, as recounted by Thucydides [75, II.37, p. 145]:

> Let me say that our system of government does not copy the institutions of our neighbours. It is more the case of our being a model to others, than of our imitating anyone else. Our constitution is called a democracy because power

[1] `http://www.un.org/en/universal-declaration-human-rights/`, accessed September 2, 2016

[2] `http://www.ohchr.org/EN/UDHR/Pages/Language.aspx?LangID=trk`, accessed September 2, 2016

> is in the hands not of a minority but of the whole people (*καὶ ὄνομα μὲν διὰ τὸ μὴ ἐς ὀλίγους ἀλλ’ ἐς πλείονας οἰκεῖν δημοκρατία κέκληται*). **When it is a question of settling private disputes, everyone is equal before the law;** when it is a question of putting one person before another in positions of public responsibility, what counts is not membership of a particular class, but the actual ability which the man possesses. No one, so long as he has it in him to be of service to the state, is kept in political obscurity because of poverty. And, just as our political life is free and open, so is our day-to-day life in our relations with each other. We do not get into a state with our next-door neighbour if he enjoys himself in his own way, nor do we give him the kind of black looks which, though they do no real harm, still do hurt people’s feelings. We are free and tolerant in our private lives; but in public affairs we keep to the law. This is because is commands our deep respect.
>
> We give our obedience to those whom we put in positions of authority, and we obey the laws themselves, especially those which are for the protection of the oppressed, and those unwritten laws which it is an acknowledged shame to break.

I don’t know if the translator is right to have Thucydides’s Pericles say that power is in the hands of the whole people. The Greek refers literally only to a majority. In Turkish, Furkan Akderin seems to translate more literally [76, p. 82]:

> Siyasi yapımızın komşularımızdan bir farkı yok. Hatta onlardan üstün olduğumuzu bile söyleyebiliriz. Çünkü biz onlara göre değil, onlar bize göre yasalarını yapıyorlar. Bizim devletimiz azınlığın değil çoğunluğun çıkarlarını gözetmektedir. Bu nedenle de ismi demokrasidir. **Herhangi bir anlaşmazlık anında herkes yasalar karşısında eşittir.** Ancak konu kamu yaşamına katılmak olduğunda kim diğerlerinden daha üstünse yönetimde o bulunur. Atina’ya hizmet eden hiç kimse kendisinin fakir ol-

> masından ya da o an için devletin bulunduğu güç durumdan dolayı utanç duymaz. Devlet içinde esas olan şey özgürlüktür. Fakat gündelik yaşamımızda özgürlükten kavramından yola çıkarak önümüze gelen herkese kötü davranmayız. Ya da bir insan maddi durumundan dolayı başkaları tarafından aşağılanmaz. Gündelik işlerimizde karşımızdaki insanları zorlayamayız. Diğer yandan memurlara, yasalara özellikle de zor durumda olanlara karşı suç işlenmesini engelleyen yasalara karşı saygı gösterilmesi ve saygı göstermeyenlerin dışlanması çok önemlidir.

In mathematics, the sign = of equality was introduced by Robert Recorde in 1557 [68] as an icon of two parallel lines having the same length,

> bicause no. 2. thynges, can be moare equalle.

The equals sign is an icon in the strict sense of Charles Peirce [60], because it

> would possess the character which renders it significant, even though its object had no existence; such as a lead-pencil streak as representing a geometrical line.

4.5. The Pythagorean Theorem

I have tried to come up with a visual proof of the Pythagorean Theorem based on Euclid's proof. The point is to be able to sketch a proof of a nontrivial theorem using only a diagram (which can be prepared in advance). Having tested Figure 4.1 on one non-mathematician, I do not know how well the layperson can see the large right triangle with a square on each side, rather than just the smaller shapes that make up these larger ones.

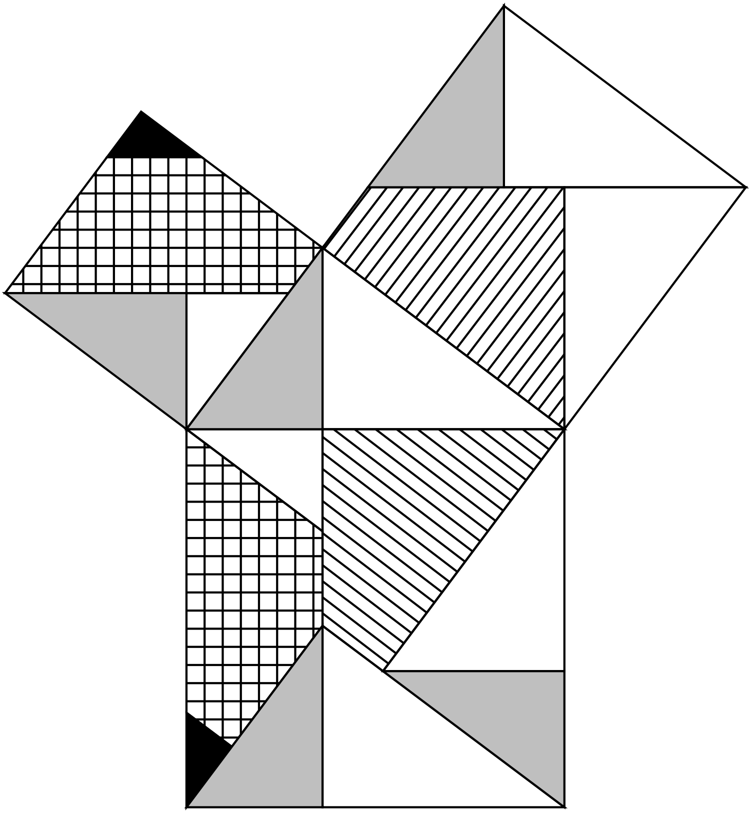


Figure 4.1.: The Pythagorean Theorem

4.6. Thales

The *Wikipedia* article on Thales needs thorough editing, at least as far as Thales's mathematics is concerned, to temper dogmatic assertions about what Thales could prove. However, the article does provide a useful reference, which is to the apophthegm of Thales quoted by Diogenes just after the two cited by Collingwood (page 38). Perhaps all of them should be given:

> Of all things that are, the most ancient is God, for he is uncreated.
> The most beautiful is the universe, for it is God's workmanship.
> The greatest is space, for it holds all things.
> The swiftest is mind, for it speeds everywhere.
> The strongest, necessity, for it masters all.
> The wisest, time, for it brings everything to light.
> *πρεσβύτατον τῶν ὄντων θεός· ἀγένητον γάρ.*
> *κάλλιστον κόσμος· ποίημα γὰρ θεοῦ.*
> *μέγιστον τόπος· ἅπαντα γὰρ χωρεῖ.*
> *τάχιστον νοῦς· διὰ παντὸς γὰρ τρέχει.*
> *ἰσχυρότατον ἀνάγκη· κρατεῖ γὰρ πάντωον.*
> *σοφώτατον χρόνος· ἀνευρίσκει γὰρ πάντα.*

The claim in *Wikipedia* (as of September 27, 2016) is,

> Topos is in Newtonian-style space, since the verb, chorei, has the connotation of yielding before things, or spreading out to make room for them, which is extension. Within this extension, things have a position. Points, lines, planes and solids related by distances and angles follow from this presumption.

Beyond the bare apophthegm about space, no justification is offered for this claim; thus I think it is original research, unfit for *Wikipedia.*

It is suggested that Thales knew about the Egyptian *seked,* the inverse of our notion of slope, except run is measured in palms; and rise, cubits; there being seven palms in the cubit.

The textbook William G. Shute, William W. Shirk, George F. Porter, *Plane and Solid Geometry,* American Book Company (1960), pp. 25-27, is cited for the claim:

> when Thales visited Egypt, he observed that whenever the Egyptians drew two intersecting lines, they would measure the vertical angles to make sure that they were equal. Thales concluded that one could prove that all vertical angles are equal if one accepted some general notions such as: all straight angles are equal, equals added to equals are equal, and equals subtracted from equals are equal.

Apparently the textbook offers a proof in the style of Euclid. My idea is that the proof would be by symmetry. This seems to be the idea of "A Mathematician's Lament" [55, pp. 18–9].

A. Program

The invitation to speak at the Thales festival on September 24, 2016, came by email as follows.

> TURAD'ın (Turizm Araştırmaları Derneği) Didim Belediyesi adına yürüttüğü "Markalaşma ve Turizmde Sürdürülebilirlik Projesi" kapsamında düzenlenen bir dizi etkinlikle, Didim'in tarihi ve kültürel değerlerini ülkemiz ve dünyada tanıtarak uzun yıllar boyunca yaşatmayı hedefliyoruz. Bu bağlamda düzenlenen I. Thales Buluşması 24 Eylül'de Thales'in memleketi Milet'te gerçekleştirilecektir.
>
> Amacımız, önümüzdeki yıllarda Thales etkinliğini uluslararası alanda bir konferansa dönüştürmek ve değerli akademisyenleri her yıl aynı tarihlerde Didim'de misafir etmektir.
>
> Konferansın yanısıra, Belediye Başkanlığı Didim'de matematiğin güncel yaşamdaki yerini benimsetecek, analitik düşünme yetkinliği kazandıracak, soran, sorgulayan bireyler yetiştirecek Thales Bilim Akademisi'ni kurmayı da hedeflemektedir. Türk Matematik Derneği Yönetim Kurulu, projemize destek vermeyi kabul etmiştir. Dernek, ilk yıl organizasyonunun bilimsel ve akademik ayağı olmayı üstlenmiş ve sizi davetli konuşmacı olarak önermiştir. Milet Anfitiyatro'da, bir konuşma yapmak üzere sizi de aramızda görmekten onur duyarız.
>
> Konuşma, siz değerli misafirlerimizin tercih edeceği bir konuda, yaklaşık 20 dakikalık sürede tamamlanacak şekilde planlanmıştır.
>
> PROGRAM
>
> - 24 Eylül Saat 12:00'de öğle yemeğinde buluşma

  - Yemekten sonra Apollon Tapınağı gezisi ve Milet'e gidiş
  - Milet Antik Kenti gezisi, anfitiyatroda konuşmalar
  - Basın açıklaması
- Milet Müzesi bahçesinde kokteyl
- 20:00 Akşam kapanış yemeği (D-Marin)
- 25 Eylül Kahvaltı ve dönüş

(Konaklama : Venosa Beach Resort Hotel)

Didim Belediyesinin evsahipliğinde gerçekleşecek bu programda, katılım teyidinizi takiben seyahat ve konaklama organizasyonunuz ekibimiz tarafından gerçekleştirilecektir.

İlginize şimdiden teşekkür eder, geri dönüşünüzü rica ederiz.

The list of speakers and titles was later announced as:

Betül Tanbay Açılış Konuşması
Ayşe Berkman Tales'in Hesapları
David Pierce Kanıt Kavramının Öncüsü Olarak Thales
Attila Aşkar Finans Piyasalarında Futures Yatırım Sistemi
Ali Karatay Tales ilk matematik felsefecisi miydi?
Alp Eden Cumhuriyet Dönemi Matematikçileri

All speakers had the title Profesör Doktor. We had been told that one talk would be called Thales'in bugünü etkileyen çalışmaları; this was apparently replaced with the talk whose title referred to finance.

B. My talk

In June 2015 in Istanbul, at the 5th World Congress and School on Universal Logic, I gave course of three lectures on the Compactness Theorem. After the course, I typed up my handwritten lecture-notes, correcting them according to my memory of what I had actually written on the whiteboards and said out loud. I needed about six typeset pages (size A5, 12 point type, text body taking nine sixteenths of the area of a page) to cover each lecture; I had covered about four handwritten pages in each lecture. Each lecture had lasted one hour.

For the 2016 Thales Meeting, I had to prepare to speak for a third of an hour. Should I distill my thoughts on Thales into two pages? I did not expect to be writing on boards, though I planned to use some models to explain some theorems. In the event, practice showed that 20 minutes were enough for six typeset pages (of the dimensions described, but including some diagrams). Including front page and contents page, I had eight pages (printed on either side of two sheets of A4 paper) to take to the Meeting.

Below, typeset as a quotation, is the English text that I had started with. In preparing the Turkish version from this, I ended up dropping

1) the paragraph about Teos;
2) the second sentence in the quotation from the Universal Declaration of Human Rights, and all of the ensuing quotation from Pericles; and

3) any mention of the rule $A = \pi r^2$.

I didn't sketch the simple Figure B.1, since the Meeting organizers had been able to fulfill my request to print out, on a foam board of size A1, the color version of Figure 4.1.

Since Thales's birth year had been given as 625 in an earlier talk, I pointed out in my own talk that this year was based on the assumption that Thales had been 40 years old in the year of the solar eclipse.

I had planned to use two chopsticks to talk about vertical angles. I had also prepared some cardboard triangles and circles. In the event, I forgot these in the hotel. At the Didim restaurant where we lunched, I was able to pick up two drinking straws to use in place of the chopsticks. At the theater of Miletus, I folded a sheet of paper into an isosceles triangle. Since a whiteboard was available, I drew something like Figure 2.6a.

I had assumed that the voices of speakers would be unamplified, as they would have been in ancient times. But we had to speak into a microphone: either the microphone fixed to the podium, which to my feeling was too far from the audience, or the handheld microphone, which kept me from using props (the drinking straws, the paper triangle) as I wished.

I had practiced my talk, but not memorized it. I ended up referred to my printed text more than I had hoped. Sometimes my words did not flow. I do not know what to do about this but continue to practice speaking Turkish.

> When I first came to Turkey, I used Herodotus as a travel guide. With Ayşe's family, when I visited the ruins of Sardis, east of Izmir, I knew that it had been the capital of Lydia. The Lydians had once been at war with the Medians for six years, when a solar eclipse occurred. This scared the warring parties into making peace. According to Herodotus, Thales

of Miletus had predicted the eclipse. We know now that the eclipse happened in 585 B.C.E.

Two weeks ago, Ayşe and I were in Teos, which is today Sığacık. In Teos there is a well-preserved bouleuterion, a council chamber. According to Herodotus, Thales recommended that the Ionians build this.

I say in my title that I want to talk about Thales as the originator of the concept of *proof.* A proof shows how a certain proposition follows from previously accepted principles. The earliest *mathematical* proofs that we still have are (mostly) those in the thirteen books of Euclid's *Elements.* These books were written around 300 B.C.E. In the mathematics department of Mimar Sinan Fine Arts University, our first-year students read the first book of the *Elements.* This culminates in two important theorems:

1. The Pythagorean Theorem: the square on the longest side (the hypotenuse) of a right triangle is equal to the squares on the other two sides.
2. Every plot of land with straight borders can be measured, in the sense of being shown equal to some rectangle on a given base.

In Figure B.1, angle ABC is right. Euclid shows that the shaded square on the left is equal to the shaded rectangle, and the blank square on the right is equal to the blank rectangle. The two rectangles together are a square. Thus the Pythagorean Theorem is proved.

Equality here is not *sameness.* That the shaded square is equal to the shaded rectangle means the two figures have the same *area*; but they are different figures.

That equality is not sameness is often forgotten in mathematics today. However, according to the Universal Declaration of Human Rights,

> All are equal before the law and are entitled without any discrimination to equal protection of the law. All are en-

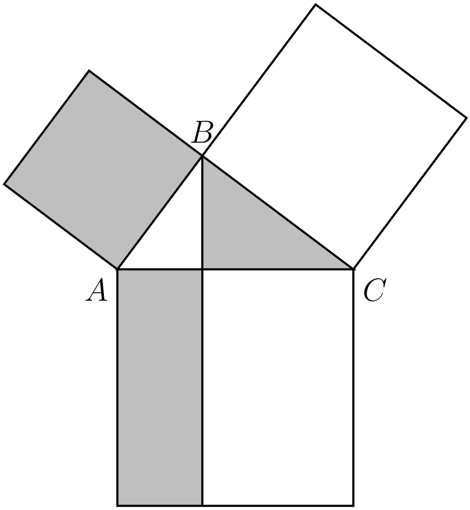


Figure B.1.: The Pythagorean Theorem

> titled to equal protection against any discrimination in violation of this Declaration and against any incitement to such discrimination.

The notion of equality before the law is traced to the Funeral Oration of Pericles in Athens, around 430 B.C.E.:

> Our constitution is called a democracy because power is in the hands not of a minority but of the whole people. When it is a question of settling private disputes, everyone is equal before the law; when it is a question of putting one person before another in positions of public responsibility, what counts is not membership of a particular class, but the actual ability which the man possesses.

In mathematics, the modern sign = of equality consists of two *equal* parallel straight lines: they are two lines, not one.

The Egyptians had already been measuring land for centuries before Euclid. Herodotus observed that the Greeks had learned mathematics from the Egyptians. And yet the

Egyptians computed the area of a four-sided field by taking the product of the averages of the opposite sides. This formula is not exact, unless the field is a rectangle. Euclid provides an exact measurement.

Perhaps Egyptian tax law defined how fields were to be measured. Our students may be given the impression that mathematics works this way. Somebody tells them a rule—say, that the area of a circle is given by

$$A = \pi r^2$$

—and they have to learn it. But this is not mathematics. An assertion that I make is not mathematics unless

1) I know it is true, and
2) I can explain why it is true.

In fact the equation $A = \pi r^2$ is a modern formulation of the most difficult theorem in Euclid's *Elements.* It is a theorem of what we now call calculus.

Several propositions in the *Elements* are said to have been known to Thales. Here are four of them:

1. A diameter of a circle bisects the circle.
2. The base angles of an isosceles triangle are equal to one another.
3. When two straight lines intersect, the vertical angles that are created are equal, each to its opposite.
4. The angle inscribed in a semicircle is right.

These are theorems about *every* circle, *every* isosceles triangle, *every* pair of intersecting straight lines. They cannot be proved by measuring specific examples. They can be understood by means of a single principle: symmetry.

[Now demonstrate, using props.]

Thales is also said to have thought the whole world was unified by a single underlying principle, which could be identified with water. In the same way, perhaps, Thales recognized that a single principle could account for every instance of the four propositions above. Identifying such principles is what mathematics is about; and in this sense, Thales may have been the first mathematician.

C. Socrates's proof of the immortality of the soul

Referred to in §4.1 (page 80), the following is from Book X of the *Republic* [63], with formatting and highlighting by me. The *Republic* is told in the first person by Socrates himself. The reason for including Socrates's argument here is that it is an example of a mathematical proof of a non-mathematical assertion.

> "Have you never perceived," said I, "that **our soul is immortal** (*ἀθάνατος ἡμῶν ἡ ψυχή*) and never perishes?"
>
> And he, looking me full in the face in amazement, said, "No, by Zeus, not I; but are you able to declare this?"
>
> "I certainly ought to be," said I, "and I think you too can, for it is nothing hard."
>
> "It is for me," he said; "and I would gladly hear from you this thing that is not hard."
>
> "Listen," said I.
>
> "Just speak on," he replied.
>
> "You speak of good [608e] and evil, do you not?"
>
> "I do."
>
> "Is your notion of them the same as mine?"
>
> "What is it?"
>
> "That which destroys and corrupts in every case is the evil; that which preserves and benefits is the good."

“Yes, I think so,” he said.

“How about this: **Do you say that there is for everything its special good and evil** (*κακὸν ἑκάστῳ τι καὶ ἀγαθὸν λέγεις*), [609a] as for example for the eyes ophthalmia, for the entire body disease, for grain mildew, rotting for wood, rust for bronze and iron, and, as I say, for practically everything its congenital evil and disease?”

“I do,” he said.

“Then when one of these evils comes to anything does it not make the thing to which it attaches itself bad, and finally disintegrate and destroy it?”

“Of course.”

“Then the congenital evil of each thing and its own vice destroys it, or **if that is not going to destroy it, nothing else** [609b] **remains that could** (*εἰ μὴ τοῦτο ἀπολεῖ, οὐκ ἂν ἄλλο γε αὐτὸ ἔτι διαφθείρειεν*); for obviously the good will never destroy anything, nor yet again will that which is neutral and neither good nor evil.”

“How could it?” he said.

“If, then, we discover anything that has an evil which vitiates it, yet is not able to dissolve and destroy it, shall we not thereupon know that of a thing so constituted there can be no destruction?”

“That seems likely,” he said.

“Well, then,” said I, “**has not the soul something that makes it evil** (*ψυχῇ ἆρ’ οὐκ ἔστιν ὃ ποιεῖ αὐτὴν κακήν*)?”

“Indeed it has,” he said, “all the things that we were just now enumerating, [609c] **injustice and licentiousness and cowardice and ignorance** (*ἀδικία τε καὶ ἀκολασία καὶ δειλία καὶ ἀμαθία*).”

“**Does any one of these things dissolve and destroy it** (*Ἦ οὖν τι τούτων αὐτὴν διαλύει τε καὶ ἀπόλλυσι*)? And reflect, lest we be misled by supposing that when an unjust and foolish man is taken in his injustice he is then destroyed by the injustice, which is the vice of soul. But conceive it thus: Just as the vice of body which is disease wastes and destroys it so that it no longer is a body at all, in like manner in all the examples of which we spoke it is the specific evil which, [609d] by attaching itself to the thing and dwelling in it with power to corrupt, reduces it to nonentity. Is not that so?"

"Yes."

"Come, then, and consider the soul in the same way. Do injustice and other wickedness dwelling in it, by their indwelling and attachment to it, corrupt and wither it till they bring it to death and separate it from the body?"

"They certainly do not do that," he said.

"But surely," said I, "it is unreasonable to suppose that the vice of something else destroys a thing while its own does not."

"Yes, unreasonable."

"For observe, Glaucon," [609e] said I, "that we do not think it proper to say of the body either that it is destroyed by the badness of foods themselves, whether it be staleness or rottenness or whatever it is; but when the badness of the foods themselves engenders in the body the defect of body, then we shall say that it is destroyed owing to these foods, but by its own vice, which is disease. [610a] But the body being one thing and the foods something else, we shall never expect the body to be destroyed by their badness, that is by an alien evil that has not produced in it the evil that belongs to it by nature."

"You are entirely right," he replied.

"On the same principle," said I, "**if the badness of the body does not produce in the soul the soul's badness we shall never expect the soul to be destroyed by an alien evil apart from its own defect** (*ἐὰν μὴ σώματος πονηρία ψυχῇ ψυχῆς πονηρίαν ἐμποιῇ, μή ποτε ἀξιῶμεν ὑπ'* ἀλλοτρίου κακοῦ ἄνευ τῆς ἰδίας πονηρίας ψυχὴν ἀπόλλυσθια)—one thing, that is, by the evil of another."

"That is reasonable," he said.

"Either, then, we must refute this [610b] and show that we are mistaken, or, so long as it remains unrefuted, we must never say that by fever or any other disease, or yet by the knife at the throat or the chopping to bits of the entire body, there is any more likelihood of the soul perishing because of these things, until it is proved that owing to these affections of the body the soul itself becomes more unjust and unholy. But when an evil of something else occurs in a different thing and the evil that belongs to the thing is not engendered in it, [610c] we must not suffer it to be said that the soul or anything else is in this way destroyed."

"But you may be sure," he said, "that nobody will ever prove this, that the souls of the dying are made more unjust by death."

"But if anyone," said I, "dares to come to grips with the argument and say, in order to avoid being forced to admit the soul's immortality, that a dying man does become more wicked and unjust, we will postulate that, if what he says is true, injustice must be fatal [610d] to its possessor as if it were a disease, and that those who catch it die because it kills them by its own inherent nature, those who have most of it quickest, and those who have less more slowly, and not, as now in fact happens, that the unjust die owing to this but by the action of others who inflict the penalty."

"Nay, by Zeus," he said, "injustice will not appear a very terrible thing after all if it is going to be fatal to its possessor, for that would be a release from all troubles. But I rather think it will prove to be quite the contrary, [610e] something that kills others when it can, but renders its possessor very lively indeed, and not only lively but wakeful, so far, I ween, does it dwell from deadliness."

“You say well,” I replied; “for when the natural vice and the evil proper to it cannot kill and destroy the soul, still less will the evil appointed for the destruction of another thing destroy the soul or anything else, except that for which it is appointed.”

“Still less indeed,” he said, “in all probability.”

“Then since it is not destroyed by any evil whatever, [611a] either its own or alien, it is evident that it must necessarily exist always, and that if it always exists it is immortal.”

“Necessarily,” he said.

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