Kümeler kuramı alıştırmaları

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Alıştırma I. Aşağıdaki bir ordinaller eşitliği her zaman doğru ise kanıtlayın; değilse bir karşıt örnek verin.

1. $\alpha + 0 = \alpha$.
2. $0 + \alpha = \alpha$.
3. $\alpha + (\beta + \gamma) = (\alpha + \beta) + \gamma$.
4. $\alpha + \beta = \beta + \alpha$.
5. $\alpha \cdot 1 = \alpha$.
6. $1 \cdot \alpha = \alpha$.
7. $\alpha \cdot 2 = \alpha + \alpha$.
8. $2 \cdot \alpha = \alpha + \alpha$.
9. $\alpha + \beta \cdot \gamma = (\alpha + \beta) \cdot \gamma$.
10. $\alpha \cdot (\beta \cdot \gamma) = (\alpha \cdot \beta) \cdot \gamma$.
11. $\alpha \cdot \beta = \beta \cdot \alpha$.
12. $\alpha \cdot (\beta + \gamma) = \alpha \cdot \beta + \alpha \cdot \gamma$.
13. $(\alpha + \beta) \cdot \gamma = \alpha \cdot \gamma + \beta \cdot \gamma$.
14. $\alpha^0 = 0$.
15. $\alpha^0 = 1$.
16. $0^\alpha = 1$.
17. $0^\alpha = 0$.
18. $\alpha^\beta = \beta^\alpha$.
19. $(\alpha + \beta)^2 = \alpha^2 + 2 \cdot \alpha \cdot \beta + \beta^2$.
20. $(\alpha + \beta)^2 = \alpha^2 + \alpha \cdot \beta + \beta \cdot \alpha + \beta^2$.

21. $\alpha^{\beta+\gamma} = \alpha^{\beta} + \alpha^{\gamma}$.
22. $(\alpha+\beta)^{\gamma} = \alpha^{\gamma} + \beta^{\gamma}$.
23. $\alpha^{\beta+\gamma} = \alpha^{\beta} \cdot \alpha^{\gamma}$.
24. $\alpha^{\beta\cdot\gamma} = (\alpha^{\beta})^{\gamma}$.
25. $(\alpha \cdot \beta)^{\gamma} = \alpha^{\gamma} \cdot \beta^{\gamma}$.
26. $\alpha^{(\beta^{\gamma})} = \left(\alpha^{\beta}\right)^{\gamma}$.

Alıştırma II. Cantor normal biçimleri bulun:

1. $1 + \omega + \omega^2 + \omega^3$.
2. $1 + \omega^2 + \omega + \omega^3$.
3. $1 + \omega^3 + \omega + \omega^2$.
4. $\omega^3 + \omega + \omega^2 + 1$.

5. $\omega^{\omega} \cdot 2 + \omega^{\omega+1} + \omega^5 \cdot 8 + \omega^{\omega} + \omega^5 + \omega \cdot 2$
6. $\omega^{\omega^{\omega}\cdot 2+\omega^{17}} \cdot 5 + \omega^{\omega^5} \cdot 14 + \omega^{\omega^{\omega}+\omega^{17}} \cdot 6 + \omega + 317$.

7. $3 \cdot (\omega + 4)$.
8. $(\omega + 4) \cdot 3$.
9. $(\omega^2 + 3) \cdot (\omega + 4)$.
10. $(\omega + 4) \cdot (\omega^2 + 3)$.
11. $(\omega^2 \cdot 5 + 3) \cdot (\omega + 4)$.
12. $(\omega + 4) \cdot (\omega^2 \cdot 5 + 3)$.

13. $(\omega^2 + \omega + 1) \cdot (\omega^3 + \omega^2 + \omega + 1)$.
14. $(\omega^2 \cdot 4 + \omega \cdot 2 + 5) \cdot (\omega^3 \cdot 16 + \omega^2 \cdot 7 + \omega \cdot 8 + 87)$.
15. $(\omega^2 \cdot 4 + \omega \cdot 2 + 5) \cdot (\omega^{\omega\cdot 3} \cdot 16 + \omega^2 \cdot 7 + \omega \cdot 8 + 87)$.
16. $(\omega^{\omega\cdot 2} \cdot 4 + \omega \cdot 2 + 5) \cdot (\omega^{\omega\cdot 3} \cdot 16 + \omega^2 \cdot 7 + \omega \cdot 8 + 87)$.
17. $(\omega^{\omega\cdot 2} \cdot 4 + \omega \cdot 2 + 5) \cdot (\omega^{\omega^3} \cdot 16 + \omega^2 \cdot 7 + \omega \cdot 8 + 87)$.

18. $(\omega + 5)^2$.
19. $9^{\omega+2}$.
20. $(\omega + 5)^{\omega+2}$.
21. $(\omega^{\omega})^{\omega^{\omega}}$.
22. $(\omega^{\omega^{\omega}})^{\omega^{\omega}}$.
23. $6^{\omega^{1330}}$.