Model-theory exercises

Math 406

2004.12.30

Problem 1. *Suppose* $\mathcal{L} = \{P\}$, *where* P *is a singulary predicate. Let* $\mathfrak{A}$ *be a* finite $\mathcal{L}$*-structure, and suppose* $\mathfrak{B} \equiv \mathfrak{A}$. *Prove* $\mathfrak{B} \cong \mathfrak{A}$.

Problem 2. *Let* $\mathcal{L} = \{P_n : n \in \omega\}$, *where each* P_n *is a singulary predicate. Let* T *be axiomatized by the sentences*

$$\begin{gathered} \forall x\ P_0 x, \\ \forall x\ (P_{n+1}x \to P_n x), \\ \exists x\ \neg(P_n x \to P_{n+1}x), \\ \forall x\ \forall y\ (\neg(P_n x \to P_{n+1}x) \to (\neg(P_n y \to P_{n+1}y) \to x = y)), \end{gathered}$$

where $n \in \omega$. *Prove that* T *is complete.*

Problem 3. *Let* Σ *and* Σ' *be sets of sentences of some* $\mathcal{L}$ *such that*

$$\Sigma \models \neg\sigma \implies \Sigma' \not\models \sigma$$

for all sentences σ *of* $\mathcal{L}$. *Prove that* $\Sigma \cup \Sigma'$ *has a model.*

Problem 4. *Suppose* Φ *and* Ψ *are* 1*-types of a consistent theory* T. *Prove that* T *has a model in which both* Φ *and* Ψ *are realized. (Use only Compactness and the appropriate definitions.)*

Problem 5. *Assuming* T *is* ω*-categorical, prove that* T *has only countably many complete types.*