First-order Compactness exercises

Math 406

2004.11.22

Problem 1. *Show that every Archimedean ordered field is elementarily equivalent to some* countable, non-Archimedean *ordered field.*

Problem 2. *Show that every non-Archimedean ordered field contains* ***infinitesimal*** *elements, that is, positive elements a that are less than every positive rational number.*

Problem 3. *Find an example of a non-Archimedean ordered field.*

Problem 4. *The* ***order*** *of an element g of a group is the size of the subgroup $\{g^n : n \in \mathbb{Z}\}$ that g generates. In a* ***periodic*** *group, all elements have finite order. Suppose G is a periodic group in which there is no finite upper bound on the orders of elements. Show that $G \equiv H$ for some non-periodic group H.*

Problem 5. *Suppose $(X, <)$ is an infinite total order in which X is* well-ordered *by $<$. Show that there is a total order $(X^*, <^*)$ such that*

$$(X, <) \equiv (X^*, <^*),$$

but X^* is not *well-ordered by $<^*$.*