# MATH 303, FINAL EXAMINATION

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**Problem 1.** In the 8th century B.C.E., the colony of Cumae (*Κύμη*) was founded, near what is now Naples, by settlers from Euboea (Eğriboz), and also from Cyme (*Κύμη*) in western Anatolia near what is now Aliağa.[1] From the Greek alphabet as used in Cumae, the Latin alphabet was ultimately derived; this came to have 23 letters:

A B C D E F G H I K L M N O P Q R S T V X Y Z.

In the year 863 C.E., a monk from Salonica named Cyril invented the so-called Glagolitic alphabet in order to translate holy scripture from Greek into Old Bulgarian. Soon after that, the simpler Cyrillic alphabet was invented.[2] After some changes (such as the abolition of a few letters by the Soviet government in 1918), the Cyrillic alphabet became the 33-letter Russian alphabet of today:

А Б В Г Д Е Ё Ж З И Й К Л М Н О П Р С Т У Ф Х Ц Ч Ш Щ Ъ Ы Ь Э Ю Я.

This alphabet retains 19 of the 24 letters of the Greek alphabet, in their original order, though not always in the original form. What *are* the 24 letters of the Greek alphabet?

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**Problem 2.** Does a square have a ratio to its side? Explain.

**Problem 3.** Suppose a magnitude $A$ has a ratio to a magnitude $B$, and a magnitude $C$ has a ratio to a magnitude $D$. What does it mean to say that $A$ has the *same* ratio to $B$ that $C$ has to $D$ (according to Definition 5 of Book V of Euclid's *Elements*)?

*Date*: Tuesday, January 12, 2010.

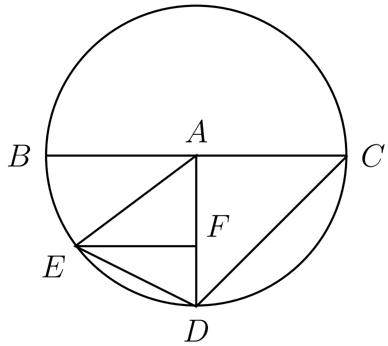
[1] Paul Harvey, *The Oxford Companion to Classical Literature* (1980); Bilge Umar, *Türkiye'deki Tarihsel Adlar* (İstanbul: İnkilâp, 1993).

[2] S. H. Gould, *Russian for the Mathematician* (Springer-Verlag, Berlin–Heidelberg–New York, 1972). Many alphabets can be seen in Carl Faulmann, *Yazı Kitabı* (Türkiye İş Bankası Kültür Yayınları, 2001).

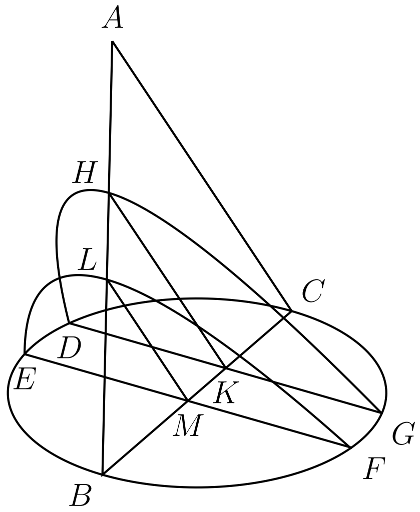
**Problem 4.** Suppose a straight line $AB$ is bisected at $C$, and another point, $D$, is chosen on $AB$. What is the relation between the squares on $AC$ and $CD$ and the rectangle contained by $AD$ and $DB$?

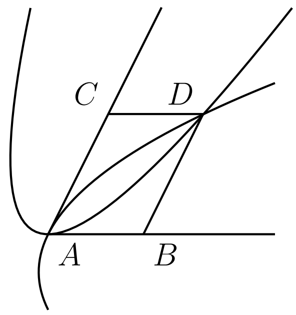
**Problem 5.** In the diagram, $BAC$ is the diameter of a circle, $A$ is the center, and $AD$ is at right angles to $BC$. Straight line $DC$ is drawn. From a point $E$ on the circumference between $B$ and $D$, the straight line $EF$ is drawn at right angles to $AD$, and $EA$ and $ED$ are drawn.

Show that the square on $DE$ has the same ratio to the square on $DC$ that the straight line $DF$ has to $DA$. (*Suggestion:* express $DE^2$ and $DC^2$ in terms of $DF$, $FA$, and $DA$.)



**Problem 6.** In the diagram on the left below, $ABC$ is an axial triangle of a cone whose base is the circle $CDEBFG$, and $DKG$ and $EMF$ are at right angles to $BC$. Planes through $DKG$ and $EMF$ cut the cone, making sections $DHG$ and $ELF$, with diameters $HK$ and $LM$, respectively; and these diameters are parallel to $AC$. The **parameters** (the 'upright sides' or *latera recta*) of the sections are not shown; but let them be $HN$ and $LP$. What is the ratio of $HN$ to $LP$ (in terms of straight lines that *are* shown in the diagram)?



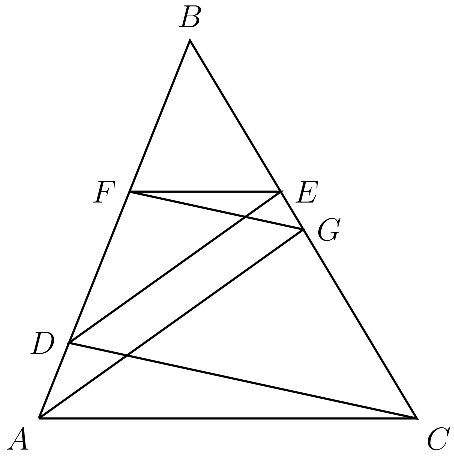

**Problem 7.** We know that an ellipse or an hyperbola has two 'conjugate' diameters, each diameter being situated ordinatewise with respect to the other. A parabola cannot have conjugate diameters in this sense. Nonetheless, suppose, in the diagram on the right above, $AB$ is the diameter of a parabola, and $AC$ is drawn ordinatewise, and $AC$ is also the diameter of another parabola, and $AB$ is situated ordinatewise with respect to $AC$. Suppose the two parabolas meet at $D$ (as well as at $A$). Let the respective ordinates $DB$ and $DC$ be dropped. Finally, suppose the parabola with diameter $AB$ has parameter $E$ (not shown), and the parabola with diameter $AC$ has parameter $F$.

Show that

$$E : AC :: AC : AB, \qquad AC : AB :: AB : F.$$

(*Remark.* It follows then that $E$ is to $F$ as the *cube* on $AC$ is to the cube on $AB$. In particular, if $E$ is twice $F$, then the cube on $AC$ is double the cube on $AB$. According to Eutocius in his *Commentary on Archimedes's Sphere and Cylinder,* Menaechmus discovered this method of 'duplicating' the cube, along with another method involving a parabola and a hyperbola. This work is the earliest known use of conic sections. For Menaechmus however, the angle $BAC$ would have been right.)

**Problem 8.** In the triangle $ABC$ below, $FG$ is parallel to $DC$, and $DE$ is parallel to $AG$. Show that $AC$ is parallel to $FE$. (You may use the theory of proportion developed in Books V and VI of the *Elements.* In that case, you will probably want to use *alternation:* if $A : B :: C : D$, then $A : C :: B : D$. You may use also that if $A : B :: E : F$ and $B : C :: D : E$, then $A : C :: D : F$. Alternatively, it is possible to avoid the theory of proportion by showing, as a lemma, that, in the diagram, $FE$ is parallel to $AC$ if and only if the parallelogram bounded by $BF$ and $BC$, in the angle $B$, is equal to the parallelogram bounded by $BE$ and $BA$. Or maybe you can find another method. In modern terms, this problem can be set in a two-dimensional vector-space; but if the scalar field of that space is non-commutative, then the claim is false.)




**Bonus.** How can this exam and this course be improved? (Responses may be submitted also by email in the next few days: `dpierce@metu.edu.tr`. Meanwhile, *iyi çalışmalar; ondan sonra, iyi tatiller!*)