**The Harnack Inequality and Littlewood-Paley Functions for Symmetric Stable Processes**

In the last 30 years, Probability has been a strong tool to study many topics in analysis. One of these topics is Littlewood-Paley theory which is useful especially in Harmonic Analysis and PDEs. Classical results regarding Littlewood-Paley theory have been proved by means of harmonic extensions where harmonic functions are defined in terms of a continuous process, namely Brownian motion.

In recent years, there has been an increasing interest in jump processes and their applications. In this talk, we consider a process which is the product of a d-dimensional symmetric stable jump process and a one-dimensional Brownian motion. By means of this process, we define $\alpha$-harmonic functions and harmonic extensions to the upper-half space $\mathbb{R}^d \times \mathbb{R}^+$. The key tool to study harmonic functions is Harnack's inequality. We discuss Harnack's inequality for $\alpha$-harmonic functions and state some regularity results. In the last part of the talk, we introduce Littlewood-Paley functions obtained by means of harmonic extension of a $\mathbb{R}^d$-function to the upper-half space $\mathbb{R}^d \times \mathbb{R}^+$. $L^p$ boundedness of these function, in the case of symmetric Markov processes, was first discussed by P.A. Meyer in late 70's. But later it was pointed out by M. Silverstein that the $L^p$-inequality fails for $p < 2$. By considering some restrictions, we obtain the $L^p$-inequality for $p > 1$. We discuss these recent results on the $L^p$-bounds of Littlewood-Paley functions.

# References

[1] D. Karlı: Harnack Inequality and Regularity for a Product of Symmetric Stable Process and Brownian Motion, Potential Analysis (2011) DOI: 10.1007/s11118-011-9265-6 , (arXiv:1010.4904v2).